Mathematics

(Chapter 9)(Differential Equations)
XII

Exercise 9.3

Question 1:

$$\frac{x}{a} + \frac{y}{b} = 1$$

Answer

$$\frac{x}{a} + \frac{y}{b} = 1$$

Differentiating both sides of the given equation with respect to x, we get:

$$\frac{1}{a} + \frac{1}{b}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b}y' = 0$$

Again, differentiating both sides with respect to x, we get:

$$0 + \frac{1}{b}y'' = 0$$

$$\Rightarrow \frac{1}{b}y'' = 0$$

$$\Rightarrow y'' = 0$$

Hence, the required differential equation of the given curve is $y'' = 0$.

Question 2:

$$y^2 = a\left(b^2 - x^2\right)$$

Answer

$$y^2 = a\left(b^2 - x^2\right)$$

Differentiating both sides with respect to x, we get:

$$2y\frac{dy}{dx} = a(-2x)$$

$$\Rightarrow 2yy' = -2ax$$

$$\Rightarrow yy' = -ax \qquad \qquad ...(1)$$

Again, differentiating both sides with respect to x, we get:

$$y' \cdot y' + yy'' = -a$$
$$\Rightarrow (y')^2 + yy'' = -a \qquad \qquad ...(2)$$

Dividing equation (2) by equation (1), we get:

$$\frac{(y')^2 + yy''}{yy'} = \frac{-a}{-ax}$$

$$\Rightarrow xyy'' + x(y')^2 - yy'' = 0$$

This is the required differential equation of the given curve.

Question 3:

$$y = ae^{3x} + be^{-2x}$$

Answer

$$y = ae^{3x} + be^{-2x} \qquad \qquad ...(1)$$

Differentiating both sides with respect to x, we get:

$$y' = 3ae^{3x} - 2be^{-2x} \qquad ...(2)$$

Again, differentiating both sides with respect to x, we get:

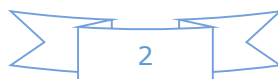$$y'' = 9ae^{3x} + 4be^{-2x} \qquad ...(3)$$

Multiplying equation (1) with (2) and then adding it to equation (2), we get:

$$(2ae^{3x} + 2be^{-2x}) + (3ae^{3x} - 2bc^{-2x}) = 2y + y'$$

$$\Rightarrow 5ae^{3x} = 2y + y'$$

$$\Rightarrow ae^{3x} = \frac{2y + y'}{5}$$

Now, multiplying equation (1) with equation (3) and subtracting equation (2) from it, we get:

$$(3ae^{3x} + 3be^{-2x}) - (3ae^{3x} - 2be^{-2x}) = 3y - y'$$

$$\Rightarrow 5be^{-2x} = 3y - y'$$

$$\Rightarrow be^{-2x} = \frac{3y - y'}{5}$$

Substituting the values of ae^{3x} and be^{-2x} in equation (3), we get:

$$y'' = 9 \cdot \frac{(2y + y')}{5} + 4\frac{(3y - y')}{5}$$

$$\Rightarrow y'' = \frac{18y + 9y'}{5} + \frac{12y - 4y'}{5}$$

$$\Rightarrow y'' = \frac{30y + 5y'}{5}$$

$$\Rightarrow y'' = 6y + y'$$

$$\Rightarrow y'' - y' - 6y = 0$$

This is the required differential equation of the given curve.

Question 4:

$$y = e^{2x}(a + bx)$$

Answer

$$y = e^{2x}(a + bx) \qquad \qquad ...(1)$$

Differentiating both sides with respect to x, we get:

$$y' = 2e^{2x}(a + bx) + e^{2x} \cdot b$$

$$\Rightarrow y' = e^{2x}(2a + 2bx + b) \qquad \qquad ...(2)$$

Multiplying equation (1) with equation (2) and then subtracting it from equation (2), we get:

$$y' - 2y = e^{2x}(2a + 2bx + b) - e^{2x}(2a + 2bx)$$

$$\Rightarrow y' - 2 = be^{2x} \qquad \qquad ...(3)$$

Differentiating both sides with respect to x, we get:

$$y''k - 2y' = 2be^{2x} \qquad \qquad ...(4)$$

Dividing equation (4) by equation (3), we get:

$$\frac{y'' - 2y'}{y' - 2y} = 2$$

$$\Rightarrow y'' - 2y' = 2y' - 4y$$

$$\Rightarrow y'' - 4y' + 4y = 0$$

This is the required differential equation of the given curve.

Question 5:

$$y = e^x (a \cos x + b \sin x)$$

Answer

$$y = e^x (a \cos x + b \sin x) \qquad \ldots(1)$$

Differentiating both sides with respect to x, we get:

$$y' = e^x (a \cos x + b \sin x) + e^x (-a \sin x + b \cos x)$$

$$\Rightarrow y' = e^x \left[(a+b) \cos x - (a-b) \sin x \right] \qquad \ldots(2)$$

Again, differentiating with respect to x, we get:

$$y'' = e^x \left[(a+b) \cos x - (a-b) \sin x \right] + e^x \left[-(a+b) \sin x - (a-b) \cos x \right]$$

$$y'' = e^x \left[2b \cos x - 2a \sin x \right]$$

$$y'' = 2e^x (b \cos x - a \sin x)$$

$$\Rightarrow \frac{y''}{2} = e^x (b \cos x - a \sin x) \qquad \ldots(3)$$

Adding equations (1) and (3), we get:

$$y + \frac{y''}{2} = e^x \left[(a+b) \cos x - (a-b) \sin x \right]$$

$$\Rightarrow y + \frac{y''}{2} = y'$$

$$\Rightarrow 2y + y'' = 2y'$$

$$\Rightarrow y'' - 2y' + 2y = 0$$

This is the required differential equation of the given curve.

Question 6:

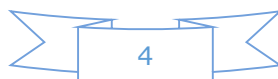Form the differential equation of the family of circles touching the y-axis at the origin.

Answer

The centre of the circle touching the y-axis at origin lies on the x-axis.

Let (a, 0) be the centre of the circle.

Since it touches the y-axis at origin, its radius is a.

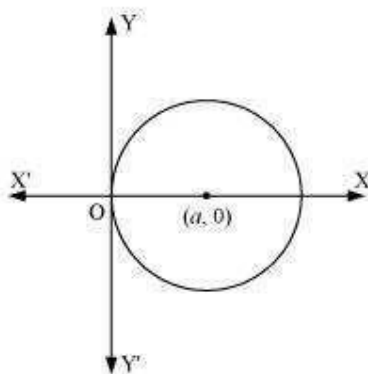Now, the equation of the circle with centre (a, 0) and radius (a) is

$$(x-a)^2 + y^2 = a^2.$$
$$\Rightarrow x^2 + y^2 = 2ax \qquad \text{...(1)}$$

Differentiating equation (1) with respect to x, we get:

$$2x + 2yy' = 2a$$
$$\Rightarrow x + yy' = a$$

Now, on substituting the value of a in equation (1), we get:

$$x^2 + y^2 = 2(x + yy')x$$
$$\Rightarrow x^2 + y^2 = 2x^2 + 2xyy'$$
$$\Rightarrow 2xyy' + x^2 = y^2$$

This is the required differential equation.

Question 7:

Form the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis.

Answer

The equation of the parabola having the vertex at origin and the axis along the positive y-axis is:
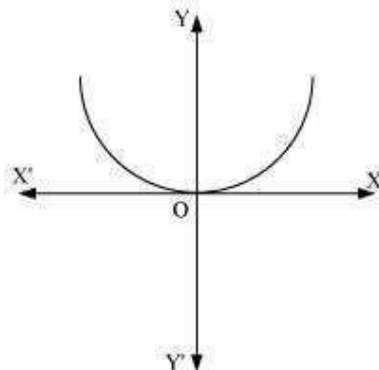
$$x^2 = 4ay \qquad \text{...(1)}$$

Differentiating equation (1) with respect to x, we get:

$$2x = 4ay' \qquad \qquad ...(2)$$

Dividing equation (2) by equation (1), we get:

$$\frac{2x}{x^2} = \frac{4ay'}{4ay}$$

$$\Rightarrow \frac{2}{x} = \frac{y'}{y}$$

$$\Rightarrow xy' = 2y$$

$$\Rightarrow xy' - 2y = 0$$

This is the required differential equation.

Question 8:

Form the differential equation of the family of ellipses having foci on y-axis and centre at origin.

Answer

The equation of the family of ellipses having foci on the y-axis and the centre at origin is as follows:
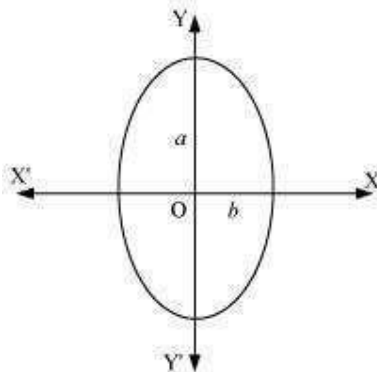
$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \qquad \qquad ..(1)$$

Free web support in Education

Differentiating equation (1) with respect to x, we get:

$$\frac{2x}{b^2} + \frac{2yy'}{b^2} = 0$$

$$\Rightarrow \frac{x}{b^2} + \frac{yy'}{a^2} = 0 \qquad \qquad ...(2)$$

Again, differentiating with respect to x, we get:

$$\frac{1}{b^2} + \frac{y' \cdot y' + y \cdot y''}{a^2} = 0$$

$$\Rightarrow \frac{1}{b^2} + \frac{1}{a^2}\left(y'^2 + yy''\right) = 0$$

$$\Rightarrow \frac{1}{b^2} = -\frac{1}{a^2}\left(y'^2 + yy''\right)$$

Substituting this value in equation (2), we get:

$$x\left[-\frac{1}{a^2}\left((y')^2 + yy''\right)\right] + \frac{yy'}{a^2} = 0$$

$$\Rightarrow -x(y')^2 - xyy'' + yy' = 0$$

$$\Rightarrow xyy'' + x(y')^2 - yy' = 0$$
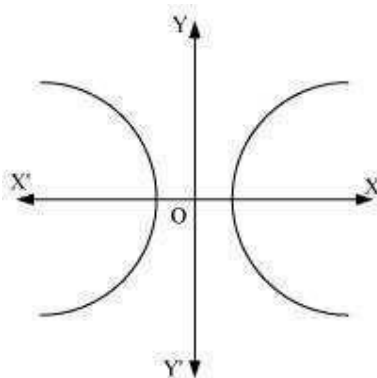
This is the required differential equation.

Question 9:

Form the differential equation of the family of hyperbolas having foci on x-axis and centre at origin.

Answer

The equation of the family of hyperbolas with the centre at origin and foci along the xaxis is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad\qquad ...(1)$$

Differentiating both sides of equation (1) with respect to x, we get:

$$\frac{2x}{a^2} - \frac{2yy'}{b^2} = 0$$

$$\Rightarrow \frac{x}{a^2} - \frac{yy'}{b^2} = 0 \qquad\qquad ...(2)$$

Again, differentiating both sides with respect to x, we get:

$$\frac{1}{a^2} - \frac{y' \cdot y' + yy''}{b^2} = 0$$

$$\Rightarrow \frac{1}{a^2} = \frac{1}{b^2}\left((y')^2 + yy''\right)$$

Substituting the value of $\frac{1}{a^2}$ in equation (2)

$$\frac{x}{b^2}\left((y')^2 + yy''\right) - \frac{yy'}{b^2} = 0$$

$$\Rightarrow x(y')^2 + xyy'' - yy' = 0$$

$$\Rightarrow xyy'' + x(y')^2 - yy' = 0$$

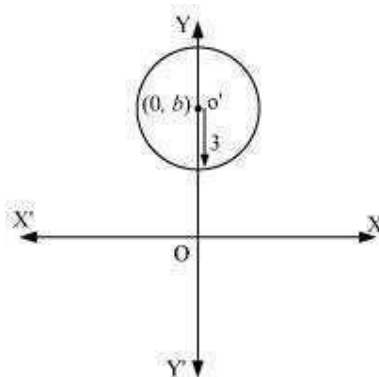This is the required differential equation.

Question 10:
Form the differential equation of the family of circles having centre on y-axis and radius 3 units.
Answer
Let the centre of the circle on y-axis be (0, b).

The differential equation of the family of circles with centre at (0, b) and radius 3 is as follows:

$$x^2 + (y-b)^2 = 3^2$$

$$\Rightarrow x^2 + (y-b)^2 = 9 \qquad \qquad ...(1)$$

Differentiating equation (1) with respect to x, we get:

$$2x + 2(y-b) \cdot y' = 0$$

$$\Rightarrow (y-b) \cdot y' = -x$$

$$\Rightarrow y - b = \frac{-x}{y'}$$

Substituting the value of (y − b) in equation (1), we get:

$$x^2 + \left(\frac{-x}{y'}\right)^2 = 9$$

$$\Rightarrow x^2 \left[1 + \frac{1}{(y')^2}\right] = 9$$

$$\Rightarrow x^2 \left((y')^2 + 1\right) = 9(y')^2$$

$$\Rightarrow (x^2 - 9)(y')^2 + x^2 = 0$$

This is the required differential equation.

Question 11:

Which of the following differential equations has $y = c_1 e^x + c_2 e^{-x}$ as the general solution?

A. $\dfrac{d^2 y}{dx^2} + y = 0$

B. $\dfrac{d^2 y}{dx^2} - y = 0$

C. $\dfrac{d^2 y}{dx^2} + 1 = 0$

D. $\dfrac{d^2 y}{dx^2} - 1 = 0$

Answer

The given equation is:

$$y = c_1 e^x + c_2 e^{-x} \qquad \ldots(1)$$

Differentiating with respect to x, we get:

$$\frac{dy}{dx} = c_1 e^x - c_2 e^{-x}$$

Again, differentiating with respect to x, we get:

$$\frac{d^2 y}{dx^2} = c_1 e^x + c_2 e^{-x}$$

$$\Rightarrow \frac{d^2 y}{dx^2} = y$$

$$\Rightarrow \frac{d^2 y}{dx^2} - y = 0$$

This is the required differential equation of the given equation of curve.

Hence, the correct answer is B.

Question 12:

Which of the following differential equation has $y = x$ as one of its particular solution?

A. $$\frac{d^2 y}{dx^2} - x^2 \frac{dy}{dx} + xy = x$$

B. $$\frac{d^2 y}{dx^2} + x \frac{dy}{dx} + xy = x$$

C. $$\frac{d^2 y}{dx^2} - x^2 \frac{dy}{dx} + xy = 0$$

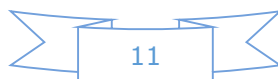D. $$\frac{d^2 y}{dx^2} + x \frac{dy}{dx} + xy = 0$$

Answer

The given equation of curve is y = x.

Differentiating with respect to x, we get:

$$\frac{dy}{dx} = 1 \qquad \qquad ...(1)$$

Again, differentiating with respect to x, we get:

$$\frac{d^2y}{dx^2} = 0 \qquad \qquad ...(2)$$

Now, on substituting the values of y, $\frac{d^2y}{dx^2}$, and $\frac{dy}{dx}$ from equation (1) and (2) in each of

the given alternatives, we find that only the differential equation given in alternative C

is correct.

$$\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0 - x^2 \cdot 1 + x \cdot x$$
$$= -x^2 + x^2$$
$$= 0$$

Hence, the correct answer is C.