Exercise 10.4

Question 1:

Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

Answer 1:

Given: Circle C (P, 3) and circle C (Q, 5) are intersecting at points A and B.

Construction: Join PA and QA. Draw PM as bisector of chord AB.

Proof: AB is chord of circle C (P, 3) and PM is bisector of chord AB.

Therefore, PM ⊥ AB

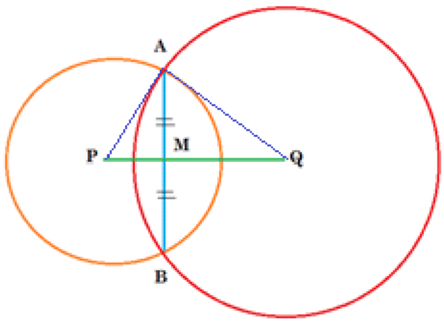
[∵ The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord]

Hence, ∠PMA = 90°

Let, PM = x, therefore, QM = 4 − x

In ΔAPM, using Pythagoras theorem

$AM^2 = AP^2 - PM^2$ ... (1)



And in ΔAPM, using Pythagoras theorem

$AM^2 = AQ^2 - QM^2$ ... (2)

From the equation (1) and (2), we get

$AP^2 - PM^2 = AQ^2 - QM^2$

$\Rightarrow 3^2 - x^2 = 5^2 - (4-x)^2 \quad \Rightarrow 9 - x^2 = 25 - (16 + x^2 - 8x)$

$\Rightarrow 9 - 9 = 8x \ \Rightarrow x = \frac{0}{8} = 0$

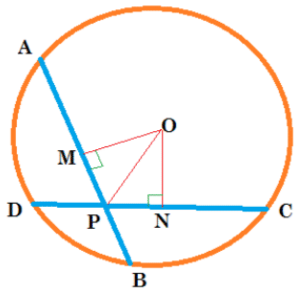
From the equation (1), $AM^2 = 3^2 - 0^2 = 9 \quad \Rightarrow AM = 3$

$\Rightarrow AB = 2AM = 6$

Question 2:

If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Answer 2:



Given: In circle C (O, r), equal chords AB and CD intersects at P.

To prove: AP = CP and BP = DP.

Construction: Join OP. Draw OM ⊥ AB and ON ⊥ CD.

Proof: In ΔOMP and ΔONP,

∠OMP = ∠ONP [∵ Each 90°]

AP = AP [∵ Common]

OM = ON [∵ Equal chords of a circle are equidistant from the centre]

Hence, ΔOMP ≅ ΔONP [∵ RHS Congruency rule]

PM = PN ... (1) [∵ CPCT]

And AB = CD ... (2) [∵ Given]

$\Rightarrow \frac{1}{2}\text{AB} = \frac{1}{2}\text{CD}$

⇒ AM = CN ... (3)

Adding the equations (1) and (3), we have

AM + PM = CN + PN

⇒ AP = CP ... (4)

Subtracting equation (4) form (2), we have

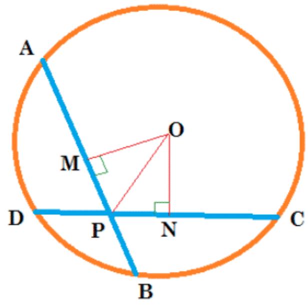
AB – AP = CD – CP

⇒ PB = PD

Question 3:

If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Answer 3:



Given: In circle C (O, r), equal chords AB and CD are intersecting at point P.

To prove: ∠OPM = ∠OPN

Construction: Join OP. Draw OM ⊥ AB and ON ⊥ CD.

Proof: In ΔOMP and ΔONP,

∠OMP = ∠ONP [∵ Each 90°]

AP = AP [∵ Common]

OM = ON [∵ Equal chords of a circle are equidistant from the centre]

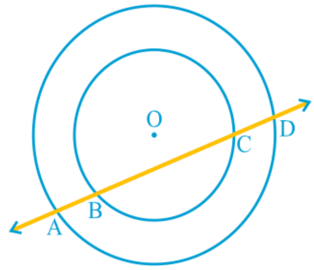
Hence, ΔOMP ≅ ΔONP [∵ RHS Congruency rule]

∠OPM = ∠OPN [∵ CPCT]

Question 4:

If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see Figure).

Answer 4:



Given: A line AB is intersecting wo concentric circles with centre O at A, B, C and D.

To prove: AB = CD.

Construction: Draw OM ⊥ AD.

Proof: BC is chord of inner circle and OM ⊥ BC. Therefore

BM = CM ... (1)

[∵ The perpendicular from the centre of a circle to a chord bisects the chord.]

Similarly, AD is chord of outer circle and OM ⊥ AD. Therefore

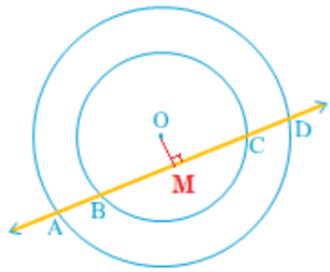
AM = DM ... (2)

[∵ The perpendicular from the centre of a circle to a chord bisects the chord.]

Subtracting the equation (1) form (2), we get

AM – BM = DM – CM

⇒ AB = CD



Question 5:

Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip?

Answer 5:

Given: In figure, points R, S and M are showing the position of Reshma, Salma and Mandeep respectively. Therefore RS = SM = 6 cm

Construction: Join OR, OS, RS, RM and OM. Draw OL ⊥ RS.

Proof: In ΔORS,

OS = OR and OL ⊥ RS [∵ By construction]

Therefore, RL = LS = 3 cm [∵ RS = 6 cm]

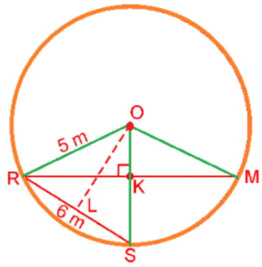
In ΔOLS, using Pythagoras theorem, $OL^2 = OS^2 - SL^2$

⇒ $OL^2 = 5^2 - 3^2 = 25 - 9 = 16$

⇒ OL = 4

In ΔORK and ΔOMK,

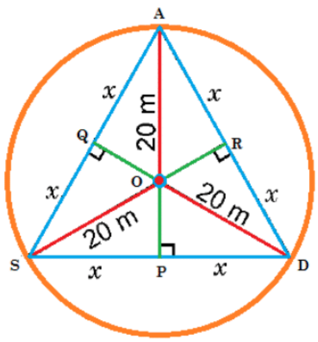
OR = OM [∵ Radii of circle]



$\angle ROK = \angle MOK$ [∵ Equal chords subtend equal angle at the centre]
$OK = OK$ [∵ Common]
Hence, $\Delta ORK \cong \Delta OMK$ [∵ SAS Congruency rule]
$RK = MK$ [∵ CPCT]
Hence, $OK \perp RM$
[∵ The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord]
Now, the area of $\Delta ORS = \frac{1}{2} \times RS \times OL$ ... (1)
And the area of $\Delta ORS = \frac{1}{2} \times OS \times KR$ ... (2)
From the equation (1) and (2),
$\frac{1}{2} \times RS \times OL = \frac{1}{2} \times OS \times KR$
$\Rightarrow RS \times OL = OS \times KR \Rightarrow 6 \times 4 = 5 \times KR \Rightarrow KR = \frac{6 \times 4}{5} = 4.8$
Hence, $RM = 2 \times KR = 2 \times 4.8 = 9.6$ cm

Question 6:
A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Answer 6:
Given: In figure, points A, S and D are the positions of Ankur, Syed and David respectively.
Therefore AS = SD = AD.
Radius of circular park = 20 m, therefore AO = SO = DO = 20
Construction: Draw $AP \perp SD$
Proof: Let $AS = SD = AD = 2x$ cm



In ΔASD,
$AS = AD$ and $AP \perp SD$ [∵ By construction]
Therefore, $SP = PD = x$ cm [∵ $SD = 2x$ cm]
In ΔOPD, using Pythagoras theorem
$OP^2 = OD^2 - PD^2$
$\Rightarrow OP^2 = 20^2 - x^2 = 400 - x^2$
$\Rightarrow OP = \sqrt{400 - x^2}$
Now, in ΔAPD, using Pythagoras theorem
$AP^2 + PD^2 = AD^2$
$\Rightarrow (AO + OP)^2 + x^2 = (2x)^2 \Rightarrow \left(20 + \sqrt{400 - x^2}\right)^2 + x^2 = 4x^2$
$\Rightarrow 400 + 400 - x^2 + 2 \times 20 \times \sqrt{400 - x^2} + x^2 = 4x^2$
$\Rightarrow 800 + 40\sqrt{400 - x^2} = 4x^2 \Rightarrow 200 + 10\sqrt{400 - x^2} = x^2$
$\Rightarrow 10\sqrt{400 - x^2} = x^2 - 200$
Squaring both sides
$100(400 - x^2) = (x^2 - 200)^2$
$\Rightarrow 40000 - 100x^2 = x^4 + 40000 - 400x^2$
$\Rightarrow x^4 - 300x^2 = 0 \Rightarrow x^2(x^2 - 300) = 0$
$\Rightarrow x^2 = 300 \Rightarrow x = 10\sqrt{3}$
Hence, the length of the string of each phone $= 2x = 2 \times 10\sqrt{3} = 20\sqrt{3}$ m