

Mathematics

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(Chapter - 12)(Heron's Formula)
(Class - 9)
Exercise 12.2

Question 1:

A park, in the shape of a quadrilateral ABCD, has $\angle C = 90^\circ$, $AB = 9$ m, $BC = 12$ m, $CD = 5$ m and $AD = 8$ m. How much area does it occupy?

Answer 1:

In quadrilateral ABCD, join BD.
In triangle BDC, by Pythagoras theorem

$$BD^2 = BC^2 + CD^2$$

$$\Rightarrow BD^2 = 12^2 + 5^2 = 144 + 25 = 169$$

$$\Rightarrow BD = 13 \text{ cm}$$

$$\text{Area of triangle BDC} = \frac{1}{2} \times BC \times DC = \frac{1}{2} \times 12 \times 5 = 30 \text{ cm}^2$$

Here, the sides of triangle ABD are $a = 9$ cm, $b = 8$ cm and $c = 13$ cm.

$$\text{So, the semi-perimeter of triangle ABD } s = \frac{a+b+c}{2} = \frac{9+8+13}{2} = \frac{30}{2} = 15 \text{ cm}$$

Therefore, using Heron's formula, area of triangle ABD = $\sqrt{s(s-a)(s-b)(s-c)}$

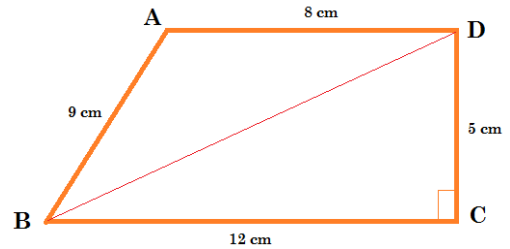
$$= \sqrt{15(15-9)(15-8)(15-13)}$$

$$= \sqrt{15(6)(7)(2)} = \sqrt{1260} = 35.5 \text{ cm}^2 \text{ (approx.)}$$

Total area of park = Area of triangle BDC + Area of triangle ABD

$$\Rightarrow \text{Total area of park} = 30 + 35.5 = 65.5 \text{ cm}^2$$

Hence, total area of park is 65.5 cm^2 .



Question 2:

Find the area of a quadrilateral ABCD in which $AB = 3$ cm, $BC = 4$ cm, $CD = 4$ cm, $DA = 5$ cm and $AC = 5$ cm.

Answer 2:

Join diagonal AC of quadrilateral ABCD.

Here, the sides of triangle ABC are $a = 3$ cm, $b = 4$ cm and $c = 5$ cm.

$$\text{So, the semi-perimeter of triangle } s = \frac{a+b+c}{2} = \frac{3+4+5}{2} = \frac{12}{2} = 6 \text{ cm}$$

Therefore, using Heron's formula, area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{6(6-3)(6-4)(6-5)} = \sqrt{6(3)(2)(1)} = \sqrt{36} = 6 \text{ cm}^2$$

And the sides of triangle ACD are $a' = 4$ cm, $b' = 5$ cm and $c' = 5$ cm.

$$\text{So, the semi-perimeter of triangle } s' = \frac{a'+b'+c'}{2} = \frac{4+5+5}{2} = \frac{14}{2} = 7 \text{ cm}$$

Therefore, using Heron's formula, area of triangle = $\sqrt{s'(s'-a')(s'-b')(s'-c')}$

$$= \sqrt{7(7-4)(7-5)(7-5)}$$

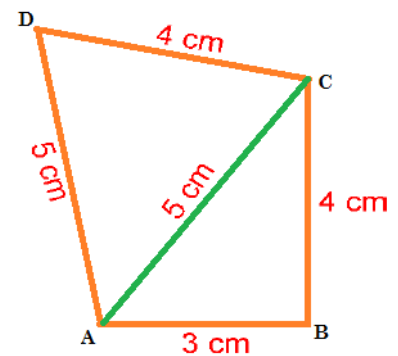
$$= \sqrt{7(3)(2)(2)}$$

$$= 2\sqrt{21} = 9.2 \text{ cm}^2 \text{ (approx.)}$$

Total area of quadrilateral = Area of triangle ABC + Area of triangle ACD

$$\Rightarrow \text{Total area of quadrilateral ABCD} = 6 + 9.2 = 15.2 \text{ cm}^2$$

Hence, the area of quadrilateral ABCD is 15.2 cm^2 .



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Question 3:

Radha made a picture of an aeroplane with coloured paper as shown in Figure. Find the total area of the paper used.

Answer 3:

For the section I:

Here, the sides of triangle are $a = 5$ cm, $b = 5$ cm and $c = 1$ cm.

So, the perimeter of triangle $s = \frac{5+5+1}{2} = \frac{11}{2} = 5.5$ cm

Therefore, using Heron's formula, area of triangle

$$= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{5.5(5.5-5)(5.5-5)(5.5-1)}$$

$$= \sqrt{5.5(0.5)(0.5)(4.5)} = \sqrt{6.1875}$$

$$= 2.5 \text{ cm}^2 \text{ (approx.)}$$

For the section II:

Here, sides of rectangle are $l = 6.5$ cm and $b = 1$ cm.

So, area of rectangle $= l \times b = 6.5 \times 1 = 6.5 \text{ cm}^2$

For the section III:

Draw $AF \parallel DC$ and $AE \perp BC$.

In quadrilateral ABCF,

$AF \parallel DC$ [\because By construction]

$AD \parallel FC$ [\because ABCD is a trapezium]

So, ABCF is a parallelogram. Therefore,

$AF = DC = 1$ cm and $AD = FC = 1$ cm [\because Opposite sides of a parallelogram]

Therefore, $BF = BC - FC = 2 - 1 = 1$ cm

\Rightarrow ABF is an equilateral triangle. [\because $AB = BF = AF = 1$ cm]

Area of equilateral triangle ABF $= \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} 1^2 = \frac{\sqrt{3}}{4}$

But, area of triangle ABF $= \frac{1}{2} \times BF \times AE$

So, $\frac{1}{2} \times BF \times AE = \frac{\sqrt{3}}{4} \Rightarrow \frac{1}{2} \times 1 \times AE = \frac{\sqrt{3}}{4}$

$\Rightarrow AE = \frac{\sqrt{3}}{2} = \frac{1.732}{2} = 0.866 = 0.9$ (approx.)

Hence, the area of trapezium ABCD $= \frac{1}{2} \times (AD + BC) \times AE$

$$= \frac{1}{2} \times (1 + 2) \times 0.9$$

$$= 1.35 = 1.4 \text{ cm}^2 \text{ (approx.)}$$

For the section IV:

Here, the base = 1.5 cm and height = 6 cm

Therefore, the area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times 1.5 \times 6 = 4.5 \text{ cm}^2$$

For the section V:

Here, the base = 1.5 cm and height = 6 cm

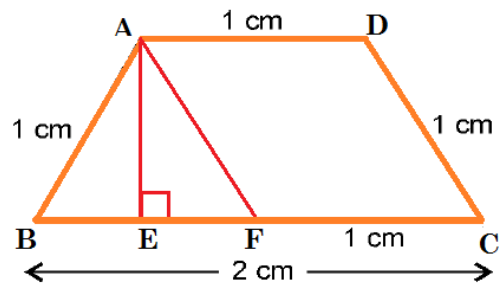
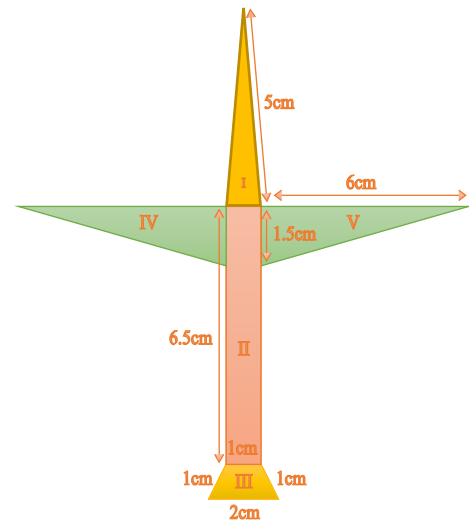
Therefore, the area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times 1.5 \times 6 = 4.5 \text{ cm}^2$$

Hence, the total area of the paper used

$=$ Area of section I $+$ Area of section II $+$ Area of section III $+$ Area of section IV $+$ Area of section V

$$= 2.5 + 6.5 + 1.4 + 4.5 + 4.5 = 19.4 \text{ cm}^2$$



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Question 4:

A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

Answer 4:

Here, the sides of triangle ABE are $a = 28$ cm, $b = 26$ cm and $c = 30$ cm.

So, the semi-perimeter of triangle $s = \frac{a+b+c}{2} = \frac{28+26+30}{2} = \frac{84}{2} = 42$ cm

Therefore, using Heron's formula, area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
 $= \sqrt{42(42-28)(42-26)(42-30)} = \sqrt{42(14)(16)(12)} = \sqrt{112896}$
 $= 336$ cm²

We know that the area of a parallelogram = base \times corresponding height

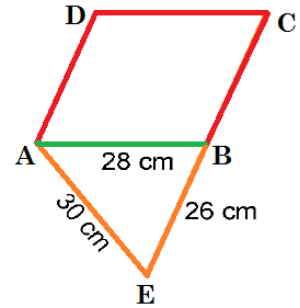
According to question:

Area of parallelogram = Area of triangle

\Rightarrow base \times corresponding height = 336

$\Rightarrow 28 \times$ corresponding height = 336

\Rightarrow corresponding height = $\frac{336}{28} = 12$ cm



Question 5:

A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?

Answer 5:

Join the diagonal AC of quadrilateral ABCD.

Here, the sides of triangle ABC are $a = 30$ m, $b = 30$ m and $c = 48$ m.

So, the semi-perimeter of triangle

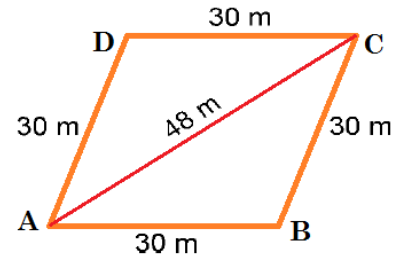
$$s = \frac{a+b+c}{2} = \frac{30+30+48}{2} = \frac{108}{2} = 54$$

Therefore, area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{54(54-30)(54-30)(54-48)} = \sqrt{54(24)(24)(6)} = \sqrt{186624} = 432$ m²

Hence, area of quadrilateral = $2 \times 432 = 864$ m²

Therefore, the area grazed by each cow = $\frac{\text{Total area}}{\text{Number of cows}} = \frac{864}{18} = 48$ m²



Question 6:

An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see Figure), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour is required for the umbrella?

Answer 6:

Here, the sides of triangle are $a = 20$ cm, $b = 50$ cm and $c = 50$ cm.

So, the semi-perimeter of triangle $s = \frac{a+b+c}{2} = \frac{20+50+50}{2} = \frac{120}{2} = 60$ cm

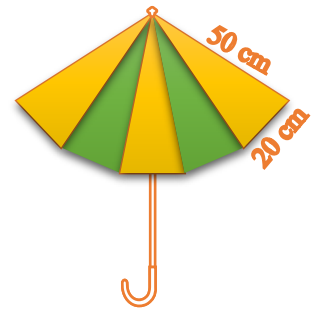
Therefore, using Heron's formula, area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{60(60-20)(60-50)(60-50)} = \sqrt{60(40)(10)(10)}$

$= 200\sqrt{6}$ cm²

So, area of 10 triangular pieces of cloths = $10 \times 200\sqrt{6} = 2000\sqrt{6}$ cm²

Hence, the area of cloths of each colour = $\frac{2000\sqrt{6}}{2} = 1000\sqrt{6}$ cm²



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Question 7:

A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in Figure. How much paper of each shade has been used in it?

Answer 7:

For Shade I:

Here, in triangle ABD, base BD = 32 cm and height AO = 16 cm.

Therefore, area of triangle ABD = $\frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times 32 \times 16$$

$$= 256 \text{ cm}^2$$

Hence, the area of paper used in shade I is 256 cm².

For Shade II:

Here, in triangle CBD, base BD = 32 cm and height CO = 16 cm.

Therefore, area of triangle CBD = $\frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times 32 \times 16$$

$$= 256 \text{ cm}^2$$

Hence, the area of paper used in shade II is 256 cm².

For Shade III:

Here, the sides of triangle CEF are $a = 6 \text{ cm}$, $b = 6 \text{ cm}$ and $c = 8 \text{ cm}$.

So, the semi-perimeter of triangle $s = \frac{a+b+c}{2} = \frac{6+6+8}{2} = \frac{20}{2} = 10 \text{ cm}$

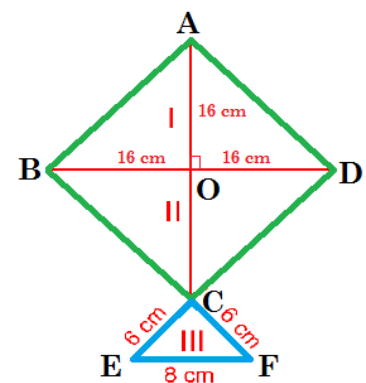
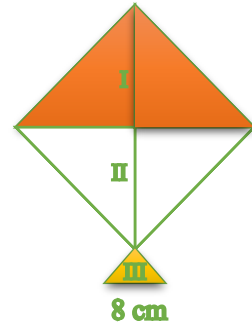
Therefore, using Heron's formula, area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{10(10-6)(10-6)(10-8)}$$

$$= \sqrt{10(4)(4)(2)}$$

$$= 8\sqrt{5} \text{ cm}^2$$

Hence, the area of paper used in shade III is $8\sqrt{5} \text{ cm}^2$.



Question 8:

A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (see Figure). Find the cost of polishing the tiles at the rate of 50p per cm².

Answer 8:

Here, the sides of triangle are $a = 9 \text{ cm}$, $b = 28 \text{ cm}$ and $c = 35 \text{ cm}$.

So, the semi-perimeter of triangle $s = \frac{a+b+c}{2} = \frac{9+28+35}{2} = \frac{72}{2} = 36 \text{ cm}$

Therefore, using Heron's formula, area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{36(36-9)(36-28)(36-35)} = \sqrt{36(27)(8)(1)}$$

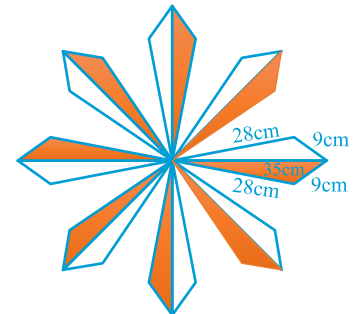
$$= \sqrt{7776}$$

$$= 88.2 \text{ cm}^2 \text{ (approx.)}$$

So, area of each triangular tile = 88.2 cm²

Therefore, area of each triangular 16 tiles = $16 \times 88.2 = 1411.2 \text{ cm}^2$

Hence, the cost of polishing the tiles at the rate of 50p per cm² = ₹ 0.50 × 1411.2 = ₹ 705.60



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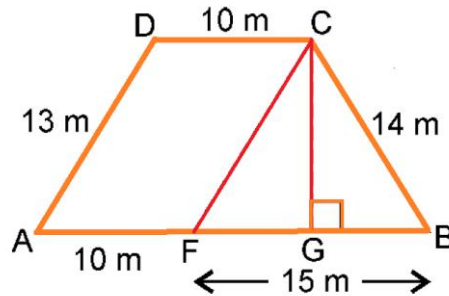
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Question 9:

A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.

Answer 9:

Draw $CF \parallel AD$ and $CG \perp AB$.



In quadrilateral ADCF,

$CF \parallel AD$ [\because By construction]

$CD \parallel AF$ [\because ABCD is a trapezium]

Therefore, ADCF is a parallelogram. So,

$AD = CF = 13$ m and $CD = AF = 10$ m [\because Opposite sides of a parallelogram]

Therefore, $BF = AB - AF = 25 - 10 = 15$ m

Here, the sides of triangle are $a = 13$ m, $b = 14$ m and $c = 15$ m.

So, the semi-perimeter of triangle

$$s = \frac{a + b + c}{2} = \frac{13 + 14 + 15}{2} = \frac{42}{2} = 21 \text{ m}$$

Therefore, using Heron's formula, area of triangle BCF = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{21(21-13)(21-14)(21-15)}$$

$$= \sqrt{21(8)(7)(6)}$$

$$= \sqrt{7056}$$

$$= 84 \text{ m}^2$$

But, the area of triangle BCF = $\frac{1}{2} \times BF \times CG$

$$\text{So, } \frac{1}{2} \times BF \times CG = 84$$

$$\Rightarrow \frac{1}{2} \times 15 \times CG = 84$$

$$\Rightarrow CG = \frac{84 \times 2}{15} = 11.2 \text{ m}$$

Therefore, area of trapezium ABCD = $\frac{1}{2} \times (AB + CD) \times CG$

$$= \frac{1}{2} \times (25 + 10) \times 11.2$$

$$= 35 \times 5.6$$

$$= 196 \text{ m}^2$$

Hence, the area of the field is 196 m^2 .