

Mathematics

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(Chapter – 2)(Polynomials)

(Class – 9)

Exercise 2.4

Question 1:

Determine which of the following polynomials has $x + 1$ a factor:

(i) $x^3 + x^2 + x + 1$

(ii) $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Answer 1:

(i) Let $p(x) = x^3 + x^2 + x + 1$

Putting $x + 1 = 0$, we get, $x = -1$

Using remainder theorem, when $p(x) = x^3 + x^2 + x + 1$ is divided by $x + 1$, remainder is given by $p(-1)$

$$= (-1)^3 + (-1)^2 + (-1) + 1$$

$$= -1 + 1 - 1 + 1$$

$$= 0$$

Since, remainder $p(-1) = 0$, hence $x + 1$ is a factor of $x^3 + x^2 + x + 1$.

(ii) Let $p(x) = x^4 + x^3 + x^2 + x + 1$

Putting $x + 1 = 0$, we get, $x = -1$

Using remainder theorem, when $p(x) = x^4 + x^3 + x^2 + x + 1$ is divided by $x + 1$, remainder is given by $p(-1)$

$$= (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$$

$$= 1 - 1 + 1 - 1 + 1$$

$$= 1$$

Since, remainder $p(-1) \neq 0$, hence $x + 1$ is not a factor of $x^4 + x^3 + x^2 + x + 1$.

(iii) Let $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

Putting $x + 1 = 0$, we get, $x = -1$

Using remainder theorem, when $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$ is divided by $x + 1$, remainder is given by $p(-1)$

$$= (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$$

$$= 1 - 3 + 3 - 1 + 1$$

$$= 1$$

Since, remainder $p(-1) \neq 0$, hence $x + 1$ is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$.

(iv) Let $p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Putting $x + 1 = 0$, we get, $x = -1$

Using remainder theorem, when $p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$ is divided by $x + 1$, remainder is given by $p(-1)$

$$= (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2}$$

$$= -1 - 1 + 2 + \sqrt{2} + \sqrt{2}$$

$$= 2\sqrt{2}$$

Since, remainder $p(-1) \neq 0$, hence $x + 1$ is not a factor of $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$.

Mathematics

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(Chapter – 2)(Polynomials)
(Class – 9)

Question 2:

Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$

(iii) $p(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$

Answer 2:

(i) $p(x) = 2x^3 + x^2 - 2x - 1$ and $g(x) = x + 1$

Putting $x + 1 = 0$, we get, $x = -1$

Using remainder theorem, when $p(x) = 2x^3 + x^2 - 2x - 1$ is divided by $g(x) = x + 1$, remainder is given by $p(-1)$

$$= (-1)^3 + (-1)^2 + (-1) + 1$$

$$= -1 + 1 - 1 + 1$$

$$= 0$$

Since, remainder $p(-1) = 0$, hence $g(x)$ is a factor of $p(x)$.

(ii) $p(x) = x^3 + 3x^2 + 3x + 1$ and $g(x) = x + 2$

Putting $x + 2 = 0$, we get, $x = -2$

Using remainder theorem, when $p(x) = x^3 + 3x^2 + 3x + 1$ is divided by $g(x) = x + 2$, remainder is given by $p(-2)$

$$= (-2)^3 + 3(-2)^2 + 3(-2) + 1$$

$$= -8 + 12 - 6 + 1$$

$$= -1$$

Since, remainder $p(-2) \neq 0$, hence $g(x)$ is not a factor of $p(x)$.

(iii) $p(x) = x^3 - 4x^2 + x + 6$ and $g(x) = x - 3$

Putting $x - 3 = 0$, we get, $x = 3$

Using remainder theorem, when $p(x) = x^3 - 4x^2 + x + 6$ is divided by $g(x) = x - 3$, remainder is given by $p(3)$

$$= (3)^3 - 4(3)^2 + (3) + 6$$

$$= 27 - 36 + 3 + 6$$

$$= 0$$

Since, remainder $p(3) = 0$, hence $g(x)$ is a factor of $p(x)$.

Question 3:

Find the value of k , if $x - 1$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = x^2 + x + k$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

(iv) $p(x) = kx^2 - 3x + k$

Answer 3:

(i) $p(x) = x^2 + x + k$

Putting $x - 1 = 0$, we get, $x = 1$

Using remainder theorem, when $p(x) = x^2 + x + k$ is divided by $x - 1$, remainder is given by $p(1)$

Mathematics

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(Chapter – 2)(Polynomials)
(Class – 9)

$$= (1)^2 + (1) + k$$
$$= 2 + k$$

Since $x - 1$ is a factor of $p(x)$, hence remainder $p(1) = 0$

$$\Rightarrow 2 + k = 0$$

$$\Rightarrow k = -2$$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

Putting $x - 1 = 0$, we get, $x = 1$

Using remainder theorem, when $p(x) = 2x^2 + kx + \sqrt{2}$ is divided by $x - 1$, remainder is given by $p(1)$

$$= 2(1)^2 + k(1) + \sqrt{2}$$

$$= 2 + k + \sqrt{2}$$

Since $x - 1$ is a factor of $p(x)$, hence remainder $p(1) = 0$

$$\Rightarrow 2 + k + \sqrt{2} = 0$$

$$\Rightarrow k = -2 - \sqrt{2}$$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

Putting $x - 1 = 0$, we get, $x = 1$

Using remainder theorem, when $p(x) = kx^2 - \sqrt{2}x + 1$ is divided by $x - 1$, remainder is given by $p(1)$

$$= k(1)^2 - \sqrt{2}(1) + 1$$

$$= k - \sqrt{2} + 1$$

Since $x - 1$ is a factor of $p(x)$, hence remainder $p(1) = 0$

$$\Rightarrow k - \sqrt{2} + 1 = 0$$

$$\Rightarrow k = \sqrt{2} - 1$$

(iv) $p(x) = kx^2 - 3x + k$

Putting $x - 1 = 0$, we get, $x = 1$

Using remainder theorem, when $p(x) = kx^2 - 3x + k$ is divided by $x - 1$, remainder is given by $p(1)$

$$= k(1)^2 - 3(1) + k$$

$$= 2k - 3$$

Since $x - 1$ is a factor of $p(x)$, hence remainder $p(1) = 0$

$$\Rightarrow 2k - 3 = 0$$

$$\Rightarrow k = \frac{3}{2}$$

Question 4:

Factorise:

(i) $12x^2 - 7x + 1$

(ii) $2x^2 + 7x + 3$

(iii) $6x^2 + 5x - 6$

(iv) $3x^2 - x - 4$

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(Chapter – 2)(Polynomials)
(Class – 9)

Answer 4:

$$\begin{aligned} \text{(i)} \quad & 12x^2 - 7x + 1 \\ &= 12x^2 - (4 + 3)x + 1 \\ &= 12x^2 - 4x - 3x + 1 \\ &= 4x(3x - 1) - 1(3x - 1) \\ &= (3x - 1)(4x - 1) \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad & 6x^2 + 5x - 6 \\ &= 6x^2 + (9 - 4)x - 6 \\ &= 6x^2 + 9x - 4x - 6 \\ &= 3x(2x + 3) - 2(2x + 3) \\ &= (2x + 3)(3x - 2) \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad & 2x^2 + 7x + 3 \\ &= 2x^2 + (6 + 1)x + 3 \\ &= 2x^2 + 6x + x + 3 \\ &= 2x(x + 3) + 1(x + 3) \\ &= (x + 3)(2x + 1) \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad & 3x^2 - x - 4 \\ &= 3x^2 - (4 - 3)x - 4 \\ &= 3x^2 - 4x + 3x - 4 \\ &= x(3x - 4) + 1(3x - 4) \\ &= (3x - 4)(x + 1) \end{aligned}$$

Question 5:

Factorise:

$$\text{(i)} \quad x^3 - 2x^2 - x + 2$$

$$\text{(iii)} \quad x^3 + 13x^2 + 32x + 20$$

$$\text{(ii)} \quad x^3 - 3x^2 - 9x - 5$$

$$\text{(iv)} \quad 2y^3 + y^2 - 2y - 1$$

Answer 5:

$$\text{(i)} \quad x^3 - 2x^2 - x + 2$$

$$\text{Let } p(x) = x^3 - 2x^2 - x + 2$$

Putting $x = 1$, we get

$$p(1) = (1)^3 - 2(1)^2 - (1) + 2 = 1 - 2 - 1 + 2 = 0$$

$\Rightarrow x - 1$ is a factor of $p(x)$.

$$\begin{array}{r|l} & x^2 - x - 2 \\ x - 1 & \overline{x^3 - 2x^2 - x + 2} \\ & \underline{x^3 - x^2} \\ & -x^2 - x + 2 \\ & \underline{-x^2 + x} \\ & + - \\ & \underline{-2x + 2} \\ & -2x + 2 \\ & \underline{+ -} \\ & 0 \end{array}$$

$$\text{So, } p(x) = (x - 1)(x^2 - x - 2)$$

$$= (x - 1)(x^2 - x - 2) = (x - 1)[x^2 - (2 - 1)x - 2]$$

$$= (x - 1)[x^2 - 2x + x - 2] = (x - 1)[x(x - 2) + 1(x - 2)]$$

$$= (x - 1)(x - 2)(x + 1)$$

$$\text{(ii)} \quad x^3 - 3x^2 - 9x - 5$$

$$\text{Let } p(x) = x^3 - 3x^2 - 9x - 5$$

Putting $x = 1$, we get

$$p(1) = (1)^3 - 3(1)^2 - 9(1) - 5 = 1 - 3 - 9 - 5 = -16 \neq 0$$

Mathematics

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(Chapter – 2)(Polynomials)

(Class – 9)

Putting $x = -1$, we get

$$p(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 5 = -1 - 3 + 9 - 5 = 0$$

$\Rightarrow x + 1$ is a factor of $p(x)$.

$$\begin{array}{r|l} x+1 & x^2 - 4x - 5 \\ & x^3 - 3x^2 - 9x - 5 \\ & x^3 + x^2 \\ & \hline & -4x^2 - 9x - 5 \\ & -4x^2 - 4x \\ & \hline & + \quad + \\ & -5x - 5 \\ & -5x - 5 \\ & \hline & + \quad + \\ & 0 \end{array}$$

So, $p(x) = (x + 1)(x^2 - 4x - 5)$

$$= (x + 1)(x^2 - 4x - 5)$$

$$= (x + 1)[x^2 - (5 - 1)x - 5]$$

$$= (x + 1)[x^2 - 5x + x - 5]$$

$$= (x + 1)[x(x - 5) + 1(x - 5)]$$

$$= (x + 1)(x - 5)(x + 1)$$

(iii) $x^3 + 13x^2 + 32x + 20$

Let $p(x) = x^3 + 13x^2 + 32x + 20$

Putting $x = -1$, we get

$$p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20 = -1 + 13 - 32 + 20 = 0$$

$\Rightarrow x + 1$ is a factor of $p(x)$.

$$\begin{array}{r|l} x+1 & x^2 + 12x + 20 \\ & x^3 + 13x^2 + 32x + 20 \\ & x^3 + x^2 \\ & \hline & 12x^2 + 32x + 20 \\ & 12x^2 + 12x \\ & \hline & - \quad - \\ & 20x + 20 \\ & 20x + 20 \\ & \hline & - \quad - \\ & 0 \end{array}$$

So, $p(x) = (x + 1)(x^2 + 12x + 20)$

$$= (x + 1)(x^2 + 12x + 20)$$

$$= (x + 1)[x^2 + (10 + 2)x + 20]$$

$$= (x + 1)[x^2 + 10x + 2x + 20]$$

$$= (x + 1)[x(x + 10) + 2(x + 10)]$$

$$= (x + 1)(x + 10)(x + 2)$$

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(Class – 9)

(iv) $2y^3 + y^2 - 2y - 1$

Let $p(y) = y^3 + y^2 - 2y - 1$

Putting $y = -1$, we get

$$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1 = -2 + 1 + 2 - 1 = 0$$

$\Rightarrow y + 1$ is a factor of $p(y)$.

$$\begin{array}{r|l} & 2y^2 - y - 1 \\ y + 1 & 2y^3 + y^2 - 2y - 1 \\ & \underline{2y^3 + 2y^2} \\ & -y^2 - 2y - 1 \\ & \underline{-y^2 - y} \\ & +y - 1 \\ & \underline{+y - 1} \\ & 0 \end{array}$$

So, $p(y) = (y + 1)(2y^2 - y - 1)$

$$= (y + 1)(2y^2 - y - 1)$$

$$= (y + 1)[2y^2 - (2 - 1)y - 1]$$

$$= (y + 1)[2y^2 - 2y + y - 1]$$

$$= (y + 1)[2y(y - 1) + 1(y - 1)]$$

$$= (y + 1)(y - 1)(2y + 1)$$

