Exercise 7.3

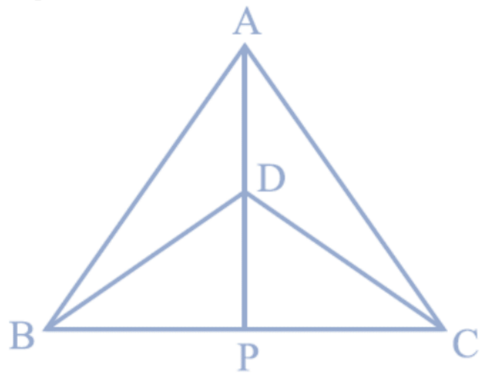
Question 1:

ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see Figure). If AD is extended to intersect BC at P, show that

(i) ΔABD ≅ ΔACD **(ii)** ΔABP ≅ ΔACP

(iii) AP bisects ∠A as well as ∠D. **(iv)** AP is the perpendicular bisector of BC.

Answer 1:



(i) In ΔABD and ΔACD,

AB = AC [∵ Given]

BD = CD [∵ Given]

AD = AD [∵ Common]

Hence, ΔABD ≅ ΔACD [∵ SSS Congruency Rule]

(ii) In ΔABD ≅ ΔACD [∵ Proved above]

∠BAD = ∠CAD [∵ CPCT]

In ΔABP and ΔACP,

AB = AC [∵ Given]

∠BAP = ∠CAP [∵ Proved above]

AP = AP [∵ Common]

Hence, ΔABP ≅ ΔACP [∵ SAS Congruency Rule]

(iii) In ΔABD ≅ ΔACD [∵ Proved above]

∠BAD = ∠CAD [∵ CPCT]

∠BDA = ∠CDA [∵ CPCT]

Hence, AP bisects both the angles A and D.

(iv) In ΔABP ≅ ΔACP [∵ Proved above]

BP = CP [∵ CPCT]

∠BPA = ∠CPA [∵ CPCT]

∠BPA + ∠CPA = 180° [∵ Linear Pair]

⇒∠CPA + ∠CPA = 180° [∵∠BPA = ∠CPA]

⇒ 2∠CPA = 180° $\Rightarrow \angle CPA = \frac{180^\circ}{2} = 90^\circ$

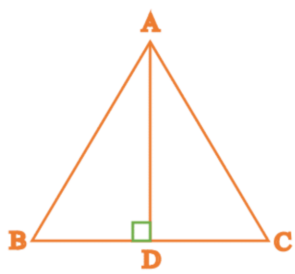
⇒ AP is perpendicular to BC. ⇒ AP is perpendicular bisector of BC. [∵ BP = CP]

Question 2:

AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that

(i) AD bisects BC **(ii)** AD bisects ∠A.

Answer 2:



(i) In ΔABD and ΔACD,

∠ADB = ∠ADC [∵ Each 90°]

AB = AC [∵ Given]

AD = AD [∵ Common]

Hence, ΔABD ≅ ΔACD [∵ RHS Congruency Rule]

BD = DC [∵ CPCT]

Hence, AD bisects BC.

(ii) ∠BAD = ∠CAD [∵ CPCT]

Hence, AD bisects angle A.

Question 3:

Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR (see Figure). Show that:

(i) ΔABM ≅ ΔPQN

(ii) ΔABC ≅ ΔPQR

Answer 3:

(i) Given that: BC = QR

$\Rightarrow \frac{1}{2}BC = \frac{1}{2}QR \;\; \Rightarrow BM = QN$ [∵ AM and PN are medians]

In ΔABM and ΔPQN,

AB = PQ [∵ Given]

AM = PN [∵ Given]

BM = QN [∵ Proved Above]

Hence, ΔABM ≅ ΔPQN [∵ SSS Congruency Rule]

(ii) In ΔABM ≅ ΔPQN [∵ Prvoed Above]

∠B = ∠Q [∵ CPCT]

In ΔABC and ΔPQR,

AB = PQ [∵ Given]

∠B = ∠Q [∵ Proved Above]

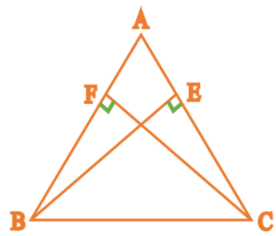
BC = QR [∵ Given]

Hence, ΔABC ≅ ΔPQR [∵ SAS Congruency Rule]

Question 4:

BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Answer 4:



In ΔFBC and ΔECB,

∠BFC = ∠CEB [∵ Each 90°]

BC = BC [∵ Common]

FC = BE [∵ Given]

Hence, ΔFBC ≅ ΔECB [∵ RHS Congruency Rule]

∠FBC = ∠ECB [∵ CPCT]

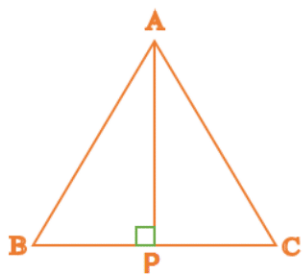
⇒ AC = AB [∵ Angles opposite to equal sides are equal]

Hence, ΔABC is an isosceles triangle.

Question 5:

ABC is an isosceles triangle with AB = AC. Draw AP ⊥ BC to show that ∠B = ∠C.

Answer 5:



In ΔABP and ΔACP,

∠APB = ∠APC [∵ Each 90°]

AB = AC [∵ Given]

AP = AP [∵ Common]

Hence, ΔABP ≅ ΔACP [∵ RHS Congruency Rule]

∠B = ∠C [∵ CPCT]