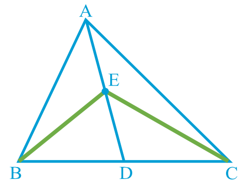
Exercise 9.3

Question 1:

In Figure, E is any point on median AD of a ΔABC. Show that ar (ABE) = ar (ACE).



Answer 1:

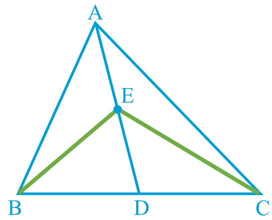
In ΔABC, AD is median. [∵ Given]
Hence, $ar(\text{ABD}) = ar(\text{ACD})$ ... (1)
[∵ A median of a triangle divides it into two triangles of equal areas.]

Similarly, in ΔEBC, ED is median. [∵ Given]
Hence, $ar(\text{EBD}) = ar(\text{ECD})$ ... (2)

Subtracting equation (2) from (1), we get
$ar(\text{ABD}) - ar(\text{EBD}) = ar(\text{ACD}) - ar(\text{ECD})$
$\Rightarrow ar(\text{ABE}) = ar(\text{ACE})$

Question 2:

In a triangle ABC, E is the mid-point of median AD. Show that $ar(\text{BED}) = \frac{1}{4}ar(\text{ABC})$.



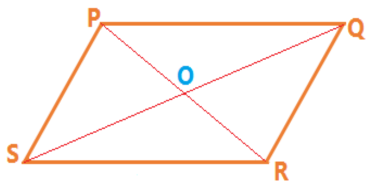
Answer 2:

In ΔABC, AD is median. [∵ Given]
Hence, $ar(\text{ABD}) = ar(\text{ACD})$
$\Rightarrow ar(\text{ABD}) = \frac{1}{2}ar(\text{ABC})$ ... (1)
[∵ A median of a triangle divides it into two triangles of equal areas.]

Similarly, in ΔABD, BE is median. [∵ E is the mid-point of AD]
Hence, $ar(\text{BED}) = ar(\text{ABE})$
$\Rightarrow ar(\text{BED}) = \frac{1}{2}ar(\text{ABD})$
$\Rightarrow ar(\text{BED}) = \frac{1}{2}\left[\frac{1}{2}ar(\text{ABC})\right]$ [∵ $ar(\text{ABD}) = \frac{1}{2}ar(\text{ABC})$]
$\Rightarrow ar(\text{BED}) = \frac{1}{4}ar(\text{ABC})$

Question 3:

Show that the diagonals of a parallelogram divide it into four triangles of equal area.



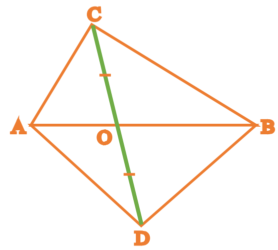
Answer 3:

Diagonals of parallelogram bisect each other.
Therefore, PO = OR and SO = OQ
In ΔPQS, PO is median. [∵ SO = OQ]
Hence, $ar(\text{PSO}) = ar(\text{PQO})$ ... (1)
[∵ A median of a triangle divides it into two triangles of equal areas.]
Similarly, in ΔPQR, QO is median. [∵ PO = OR]
Hence, $ar(\text{PQO}) = ar(\text{QRO})$ ... (2)
And in ΔQRS, RO is median. [∵ SO = OQ]
Hence, $ar(\text{QRO}) = ar(\text{RSO})$ ... (3)
From the equations (1), (2) and (3), we get
$ar(\text{PSO}) = ar(\text{PQO}) = ar(\text{QRO}) = ar(\text{RSO})$

Hence, in parallelogram PQRS, diagonals PR and QS divide it into four triangles in equal area.

Question 4:

In Figure, ABC and ABD are two triangles on the same base AB. If line- segment CD is bisected by AB at O, show that ar(ABC) = ar (ABD).



Answer 4:

In ΔADC, AO is median. [∵ CO = OD]
Hence, $ar(\text{ACO}) = ar(\text{ADO})$ ... (1)
[∵ A median of a triangle divides it into two triangles of equal areas.]

Similarly, in ΔBDC, BO is median. [∵ CO = OD]
Hence, $ar(\text{BCO}) = ar(\text{BDO})$ ... (2)

Adding equation (1) and (2), we get

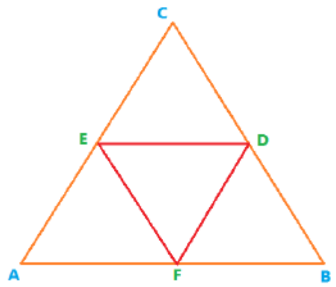
$ar(\text{ACO}) + ar(\text{BCO}) = ar(\text{ADO}) + ar(\text{BDO})$

$\Rightarrow ar(\text{ABC}) = ar(\text{ABD})$

Question 5:

D, E and F are respectively the mid-points of the sides BC, CA and AB of a Δ ABC. Show that
(i) BDEF is a parallelogram. **(ii)** $ar(\text{DEF}) = \frac{1}{4} ar(\text{ABC})$ **(iii)** $ar(\text{BDEF}) = \frac{1}{2} ar(\text{ABC})$

Answer 5:



(i) In ΔABC, E and D are mid-points of CA and BC respectively।
Hence, ED || AB and ED $= \frac{1}{2}$AB [∵ Mid-point theorem]
⇒ ED || AB and ED = FB [∵ F is mid-point of AB]
⇒ BDEF is a parallelogram.

(ii) BDEF is a parallelogram. [∵ Proved above]
$ar(\text{DEF}) = ar(\text{BDF})$ ... (1)

[∵ Diagonal of a parallelogram divide it into two triangles, equal in area]

Similarly,

AEDF is a parallelogram.
$ar(\text{DEF}) = ar(\text{AEF})$ ... (2)

तथा AEDF is a parallelogram.
$ar(\text{DEF}) = ar(\text{CDE})$ ... (3)

From the equation (1), (2) and (3), we get

$ar(\text{DEF}) = ar(\text{BDF}) = ar(\text{AEF}) = ar(\text{CDF})$

Let $ar(\text{DEF}) = ar(\text{BDF}) = ar(\text{AEF}) = ar(\text{CDF}) = x$
Therefore, $ar(\text{ABC}) = ar(\text{DEF}) + ar(\text{BDF}) + ar(\text{AEF}) + ar(\text{CDF})$

$\Rightarrow ar(\text{ABC}) = x + x + x + x = 4x = 4ar(\text{DEF})$

$$\Rightarrow ar(\text{DEF}) = \frac{1}{4} ar(\text{ABC})$$

(iii) $ar(\text{BDEF}) = ar(\text{DEF}) + ar(\text{BDF}) = x + x = 2x$

$\Rightarrow ar(\text{BDEF}) = \frac{1}{2} \times 4x$

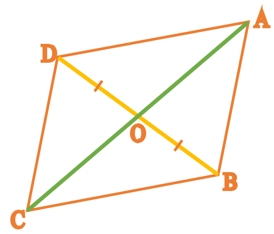
$\Rightarrow ar(\text{BDEF}) = \frac{1}{2} \times ar(\text{ABC})$ [∵ $ar(\text{ABC}) = 4x$]

Question 6:

In Figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, then show that:

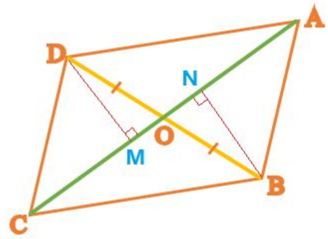
(i) ar (DOC) = ar (AOB) **(ii)** ar (DCB) = ar (ACB) **(iii)** DA || CB or ABCD is a parallelogram.

[*Hint*: From D and B, draw perpendiculars to AC.]



Answer 6:

(i) Construction: Draw perpendiculars DM and BN form D and B respectively to AC.



In ΔDMO and ΔBNO,
∠DMO = ∠BNO [∵ Each 90°]
∠DOM = ∠BON [∵ Vertically opposite angles]
DO = BO [∵ Given]
Hence, ΔDMO ≅ΔBNO [∵ AAS Congruency rule]
DM = BN ... (1) [∵ CPCT]
And $ar(\text{DMO}) = ar(\text{BNO})$ ... (2) [∵ CPCT]

In ΔDMC and ΔBNA,
∠DMC = ∠BNA [∵ Each 90°]
DM = BN [∵ From the equation (1)]
CD = AB [∵ Given]
Hence, ΔDMC ≅ΔBNA [∵ RHS Congruency rule]
And $ar(\text{DMC}) = ar(\text{BNA})$ ... (3) [∵ Congruent triangles area equal in area]
Adding the equation (2) and (3), we get

$ar(\text{DMO}) + ar(\text{DMC}) = ar(\text{BNO}) + ar(\text{BNA})$

$\Rightarrow ar(\text{DOC}) = ar(\text{AOB})$

(ii) $ar(\text{DOC}) = ar(\text{AOB})$ [∵ Proved above]
Adding $ar(\text{BOC})$ both sides

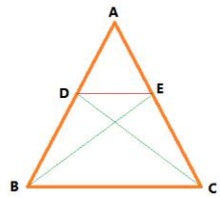
$ar(\text{DOC}) + ar(\text{BOC}) = ar(\text{AOB}) + ar(\text{BOC})$

$\Rightarrow ar(\text{DCB}) = ar(\text{ACB})$

(iii) ΔDMC ≅ΔBNA [∵ Proved above]
∠DCM = ∠BAN [∵ CPCT]
Here, the alternate angles (∠DCM = ∠BAN) are equal, Hence,
CD || AB
And CD = AB [∵ Given]
Therefore, ABCD is a parallelogram.

Question 7:

D and E are points on sides AB and AC respectively of ΔABC such that ar (DBC) = ar (EBC). Prove that DE||BC.

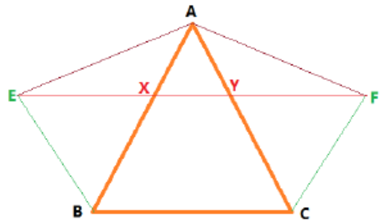


Answer 7:

ΔDBC and ΔEBC are on the same base BC and $ar(\text{DBC}) = ar(\text{EBC})$.
Therefore, DE || BC
[∵Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.]

Question 8:

XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and F respectively, show that: $ar(\text{ABE}) = ar(\text{ACF})$



Answer 8:

In quadrilateral BCYE, BE || CY [∵ BE || AC]

BC || EY [∵ BC || XY]

Therefore, BCYE is a parallelogram.

Triangle ABE and parallelogram BCYE are on the same base BE and between same parallels, BE || AC.

Hence, $ar(\text{ABE}) = \frac{1}{2} ar(\text{BCYE})$ ... (1)

[∵If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.]

Similarly, triangle ACF and parallelogram BCFX are on the same base CF and between same parallels CF || AB.

Hence, $ar(\text{ACF}) = \frac{1}{2} ar(\text{BCFX})$ ... (2)

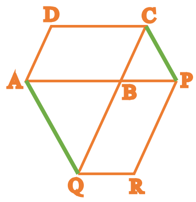
And, $ar(\text{BCYE}) = ar(\text{BCFX})$ ... (3)

[∵ On the same base (BC) and between same parallels (BC || EF), area of parallelograms are equal]

From the equation (1), (2) and (3), $ar(\text{ABE}) = ar(\text{ACF})$

Question 9:

The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see Figure). Show that ar (ABCD) = ar (PBQR).
[*Hint*: Join AC and PQ. Now compare ar (ACQ) and ar (APQ).]



Answer 9:

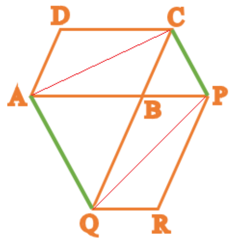
Construction: Join AC and PQ.

Triangles ACQ and APQ lie on the same base AQ and between same parallels, AQ || CP.

Hence, $ar(\text{ACQ}) = ar(\text{APQ})$

[∵ Triangles on the same base (or equal) and between the same parallels are equal in area.]

Subtracting $ar(\text{ABQ})$ from both the sides



$ar(\text{ACQ}) - ar(\text{ABQ}) = ar(\text{APQ}) - ar(\text{ABQ})$

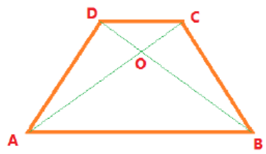
$\Rightarrow ar(\text{ABC}) = ar(\text{PBQ}) \Rightarrow \frac{1}{2} ar(\text{ABCD}) = \frac{1}{2} ar(\text{PBQR})$

[∵ Diagonal divides the parallelogram in two triangles equal in area]

$\Rightarrow ar(\text{ABCD}) = ar(\text{PBQR})$

Question 10:

Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O.
Prove that ar (AOD) = ar (BOC).



Answer 10:

Triangles ABD and ABC are on the same base AB and between same parallels, AB || CD.

Hence, $ar(\text{ABD}) = ar(\text{ABC})$

[∵ Triangles on the same base (or equal bases) and between the same parallels are equal in area.]

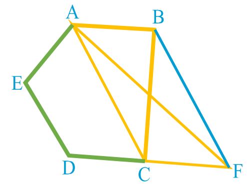
Subtracting $ar(\text{ABO})$ form both the sides

$ar(\text{ABD}) - ar(\text{ABO}) = ar(\text{ABC}) - ar(\text{ABO})$

$\Rightarrow ar(\text{AOD}) = ar(\text{BOC})$

Question 11:

In Figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that
(i) $ar(\text{ACB}) = ar(\text{ACF})$ **(ii)** $ar(\text{AEDF}) = ar(\text{ABCDE})$



Answer 11:

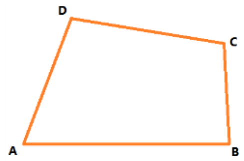
(i) Triangles ACB and ACF are on the same base AC and between same parallels AC || FB.
Hence, $ar(\text{ACB}) = ar(\text{ACF})$
[∵ Triangles on the same base (or equal bases) and between the same parallels are equal in area.]

(ii) $ar(\text{ACB}) = ar(\text{ACF})$ [∵ Proved above]
Adding $ar(\text{AEDC})$ both the sides
$ar(\text{ACB}) + ar(\text{AEDC}) = ar(\text{ACF}) + ar(\text{AEDC})$
$\Rightarrow ar(\text{ABCDE}) = ar(\text{AEDF})$

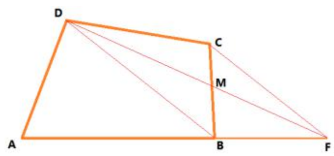
Question 12:

A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

Answer 12:

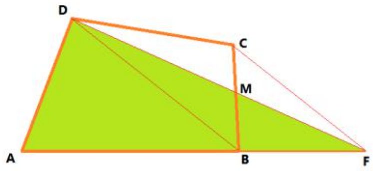


Let ABCD be the Itwaari's plot.
Join BD and through C draw a line CF parallel to BD which meet AB produced at F.
Now join D and F.
Triangles CBD and FBD are on the same base BD and between same parallels BD || CF.
Hence, $ar(\text{CBD}) = ar(\text{FBD})$
[∵ Triangles on the same base (or equal bases) and between the same parallels are equal in area.]



Subtracting $ar(\text{BDM})$ from both the sides
$ar(\text{CBD}) - ar(\text{BDM}) = ar(\text{FBD}) - ar(\text{BDM})$

$\Rightarrow ar(\text{CMD}) = ar(\text{BFM})$

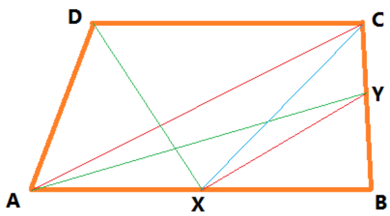


Hence, in place of ΔCMD, if ΔBFM be given to Itwaari, his plot become triangular (ΔADF).

Question 13:

ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar (ADX) = ar (ACY). [*Hint*: Join CX.]

Answer 13:



Construction: Join CX.
Triangles ADX and ACX are on the same base AX and between same parallels AB || DC.

Hence, $ar(\text{ADX}) = ar(\text{ACX})$ ... (1)

[∵ Triangles on the same base (or equal bases) and between the same parallels are equal in area.]

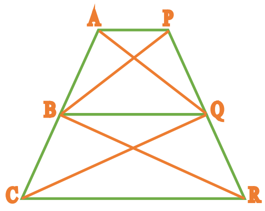
Similarly, triangles ACY and ACX are on the same base AC and between same parallels AC || XY.

Hence, $ar(\text{ACY}) = ar(\text{ACX})$ ... (2)

From the equation (1) and (2), $ar(\text{ADX}) = ar(\text{ACY})$

Question 14:

In Figure, AP || BQ || CR. Prove that ar (AQC) = ar (PBR).

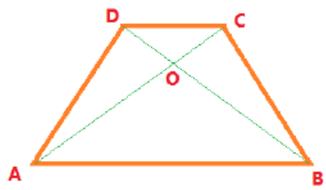


Answer 14:

Triangles ABQ and PBQ are on the same base BQ and between same parallels BQ || AP.
Hence, $ar(\text{ABQ}) = ar(\text{PBQ})$ ... (1)
[∵ Triangles on the same base (or equal bases) and between the same parallels are equal in area.]
Similarly,
Triangles BQC and BQR are on the same base BQ and between same parallels BQ || CR.
Hence, $ar(\text{BQC}) = ar(\text{BQR})$ ... (2)
Adding equation (1) and (2), we get
$ar(\text{ABQ}) + ar(\text{BQC}) = ar(\text{PBQ}) + ar(\text{BQR})$
$\Rightarrow ar(\text{AQC}) = ar(\text{PBR})$

Question 15:

Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (AOD) = ar (BOC). Prove that ABCD is a trapezium.

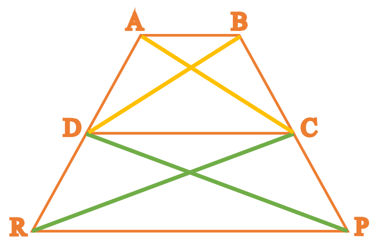


Answer 15:

$ar(\text{AOD}) = ar(\text{BOC})$ [∵ Given]
Adding $ar(\text{AOB})$ both the sides
$ar(\text{AOD}) + ar(\text{AOB}) = ar(\text{BOC}) + ar(\text{AOB})$
$\Rightarrow ar(\text{ABD}) = ar(\text{ABC})$
ΔABD and ΔABC are on the same base AB and $ar(\text{ABD}) = ar(\text{ABC})$.
Therefore, AB || DC
[∵Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.]
Hence, ABCD is a trapezium.

Question 16:

In Figure, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.



Answer 16:

$ar(\text{DRC}) = ar(\text{DPC})$ ... (1) [∵ Given]
ΔDRC and ΔDPC are on the same base DC and $ar(\text{DRC}) = ar(\text{DPC})$.
Therefore, DC || RP

[∵ Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.]
Hence, DCPR is a trapezium.

And $ar(\text{ARC}) = ar(\text{BDP})$ ... (2) [∵ Given]

Subtracting equation (1) form equation (2), we get
$ar(\text{ARC}) - ar(\text{DRC}) = ar(\text{BDP}) - ar(\text{DPC})$

$\Rightarrow ar(\text{ADC}) = ar(\text{BDC})$

ΔADC and ΔBDC are on the same base DC and $ar(\text{ADC}) = ar(\text{BDC})$.

Therefore, AB || DC

[∵ Triangles on the same base (or equal bases) and having equal areas lie between the same parallels.]
Hence, ABCD is a trapezium.