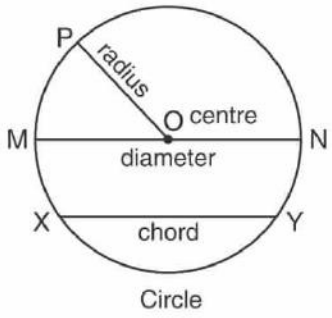
Chapter 11
Perimeter and Area

In class IV, we have learnt about a circle. Let us first revise.

A circle is a simple closed curve, every point of which is equidistant from a fixed point. This fixed point is called the centre of the circle. There are various examples of a circle around us such as the wheels of vehicles, coins, disc, the full moon, the sun etc.

TERMS ASSOCIATED WITH A CIRCLE



Circle

1. **Radius :** The distance between the centre and any point on the circle is called radius. In other words, radius is a line segment drawn from the centre to any point on the circle. The line segment OP is a radius of the circle. The line segments OM and ON are also radii of the circle.

   We can draw an infinite number of radii of a circle.

2. **Chord :** A line segment that joins any two points on a circle is called a chord of the circle. Line segments XY and MN are chords of the circle.

3. **Diameter :** The line segment drawn through the centre that joins two points on a circle is called a diameter of the circle. The line segment MN is a diameter of the circle.

4. **Circumference :** The length of the closed curve forming a circle is called its circumference.

DRAWING CIRCLES

There are two ways of drawing a circle:

1. With the help of a circular object
2. With the help of compasses

1. With the help of a circular object

We have learnt this method in the previous class. To draw a circle, we put an object like a bangle, 1 rupee coin, 5 rupee coin, a bottle cap on the plane of paper and move the pencil around it.

2. With the help of a compass

We can draw a circle of given radius with the help of compasses.


Compass

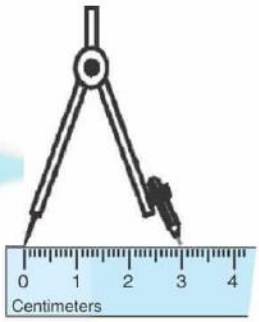
For example, let us draw a circle of radius 2.4 cm.

To draw the circle of a given measure, (here 2.4 cm) we take the following steps:

Step 1. Fix a sharp pencil firmly in the pencil holding arm of the compass. If needed tighten the pencil with the help of a screw [Fig. (a)].


(a)

Step 2. Open the compass so that its needle like arm point and the pencil tip are 2.4 cm apart [Fig. (b)].

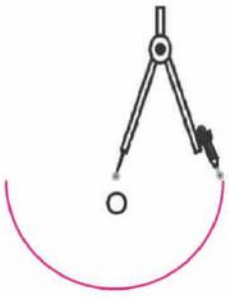


(b)

Step 3. Mark a point say, O on the sheet of a paper [Fig. (c)].

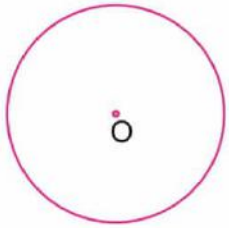
(c)

Step 4. Place the point of the needle like arm of the compass at O and move the pencil point around by holding the compass from the top until the complete circle is drawn [Fig. (d)].



(d)

Step 5. Lift the compasses. We get a circle of the given radius 2.4 cm [Fig. (e)].

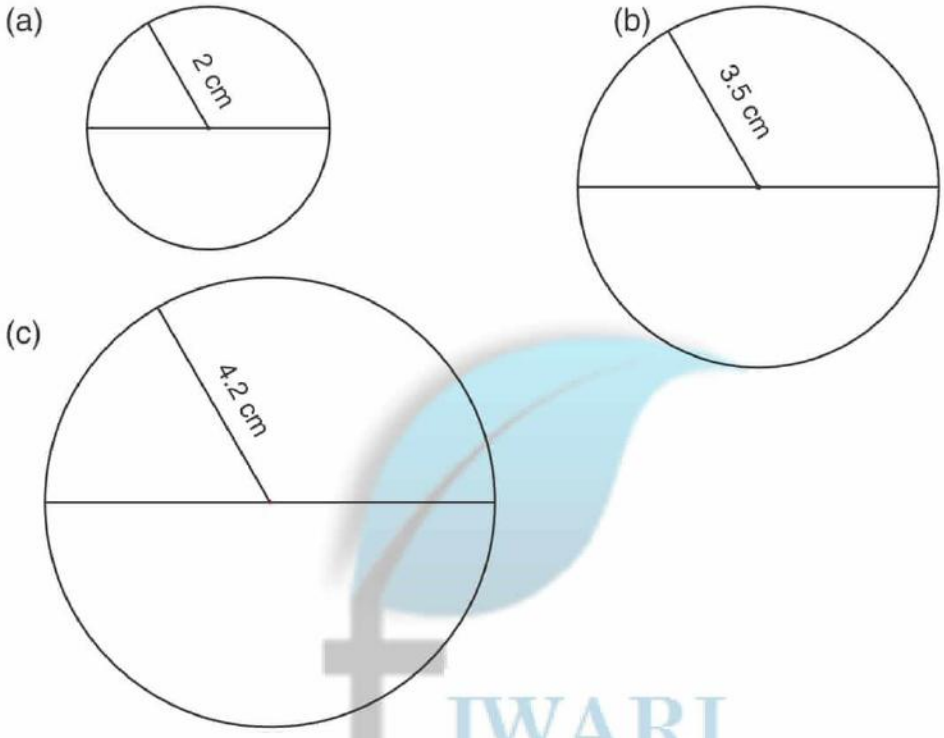


(e)

RELATIONSHIP BETWEEN RADIUS AND DIAMETER OF A CIRCLE

Let us perform the following activity, to find the relationship between radius and diameter of a circle.

Activity 1 : Draw three circles of different radii, 2 cm, 3.5 cm and 4.2 cm. In each circle, draw a radius and a diameter.



Measure the lengths of diameters of the circles. Note down the measures and fill these in a table as given below:

Circle	Length of radius	Length of diameter	Relationship between the length of radius and length of diameter
(a)	2 cm	4 cm	4 cm = 2 × 2 cm
(b)	3.5 cm	7 cm	7 cm = 2 × 3.5 cm
(c)	4.2 cm	8.4 cm	8.4 cm = 2 × 4.2 cm

What do you conclude?

We conclude that the length of the diameter of a circle is twice the length of its radius.

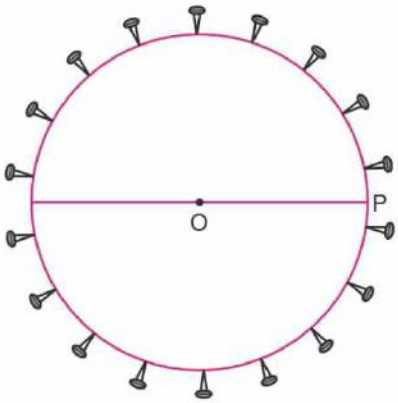
Thus,

Length of the diameter = 2 × length of the radius

RELATIONSHIP BETWEEN DIAMETER AND CIRCUMFERENCE OF A CIRCLE

Let us perform the following activity to find the relationship between diameter and circumference of a circle.

Activity 2 : Take a soft cardboard, a thread and some drawing pins. Draw a circle on the cardboard with centre O of any radius, say 3.1 cm. Fix up the pins on the circle as shown in the figure.



Fix one end of the thread with any pin and wrap the thread around the pins fixed on the circle till you reach the same pin where the thread was fixed. Now, cut the thread at the point where it touches the first pin again. Measure the length of the thread obtained after cutting it.

The length of this thread is the perimeter or circumference of the circle. You will find this length approximately 19 cm.

Thus, the circumference of the circle is approximately 19 cm.

The diameter of the circle = 2×3.1 cm = 6.2 cm

Now, $\frac{19}{6.2} = 3$ (approximately).

Repeat this activity by drawing circles of different radii. You will find the same result.

What do you conclude?

We conclude that the circumference of a circle is 3 times (approximately) the diameter of the circle.

Thus, we obtain the relation as given under:

Circumference = 3 × diameter (approx.)

Example 1 : Find the diameter of a circle of radius 2.7cm.

Solution : We have, Diameter = 2 × radius = 2 × 2.7 cm = 5.4 cm.

Example 2 : Find the radius of a circle whose diameter is 13.2 cm.

Solution : We have, Diameter = 2 × radius

It means radius = $\frac{1}{2}$ of the diameter

Hence, radius = $\frac{1}{2}$ × 13.2 cm = 6.6 cm.

Example 3 : Find the circumference (approx) of a circle whose diameter is 8 cm.

Solution : We know that, circumference = 3 × diameter (approx.)

∴ Required circumference = 3 × 8 cm

= 24 cm (approx.)

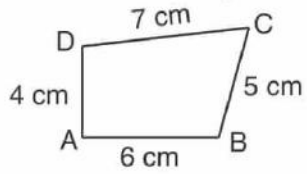
Testing Time 11.1

1. Define radius and circumference of a circle.
2. How many radii can you draw in a circle?
3. **Draw circles of the following radii:**
   (a) 2 cm (b) 2.8 cm (c) 4.1 cm
   (d) 5.7 cm
4. Draw a line segment AB of length 4 cm. Draw circles with A and B as centres with radii 4 cm in each case.
5. **Find the diametres of the circles whose radii are:**
   (a) 3 cm (b) 4.5 cm (c) 7.1 cm
   (d) 9.6 cm
6. **Find the radii of the circles whose diameters are:**
   (a) 4 cm (b) 6.2 cm (c) 8.4 cm
   (d) 14.8 cm
7. **Find the circumferences of the circles whose diameters are:**
   (a) 3 cm (b) 5 cm (c) 6.5 cm
   (d) 12.3 cm
8. **Write 'T' for true and 'F' for false statements :**
   (a) A circle is a simple closed curve. ☐
   (b) The circumference of a circle is three times (approx.) its radius. ☐
   (c) Radius is the chord of a circle. ☐
   (d) The diameter of a circle is twice its radius. ☐

Perimeter, Capacity (Volume)

PERIMETER

The perimeter of a closed figure made up of line segments is equal to the sum of the lengths of all the sides of the figure.



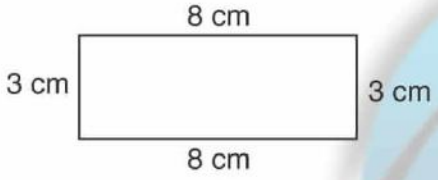
The figure given on the left side is a closed figure made up of line segments AB, BC, CD and DA. The length of all the sides are given.

To find the perimeter of the figure, we add the lengths of all sides.

$\therefore$ Perimeter of figure ABCD = AB + BC + CD + DA = 6 cm + 5 cm + 7 cm + 4 cm = 22 cm.

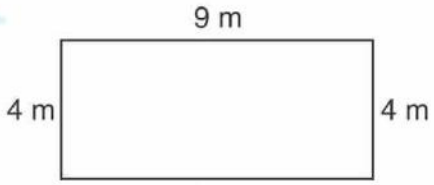
PERIMETER OF A RECTANGLE

Consider the following rectangles:



(i)

(ii)



(iii)

Find the perimeter of each of these rectangles and note down your observations in the form of a table as given below:

Rectangle	Length	Breadth	Perimeter
(i)	8 cm	3 cm	8 cm + 3 cm + 8 cm + 3 cm = 2 × 8 cm + 2 × 3 cm = 2 (8 + 3) cm
(ii)	6 cm	2 cm	6 cm + 2 cm + 6 cm + 2 cm = 2 × 6 cm + 2 × 2 cm = 2 (6 + 2) cm
(iii)	9 m	4 m	9 m + 4 m + 9 m + 4 m = 2 × 9 m + 2 × 4 m = 2 (9 + 4) m

What do you conclude?

We conclude that the perimeter of a rectangle = 2 × length + 2 × breadth = 2 × (length + breadth)

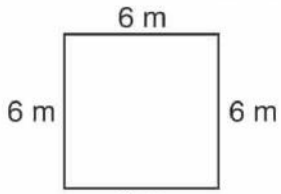
If length, breadth and perimeter are denoted by l, b and p respectively, then

$$P = 2 \times (l + b)$$

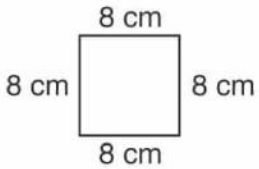
also $l = \left(\frac{P}{2} - b\right)$ and $b = \left(\frac{P}{2} - l\right)$.

PERIMETER OF A SQUARE

Consider the following squares:



(i)



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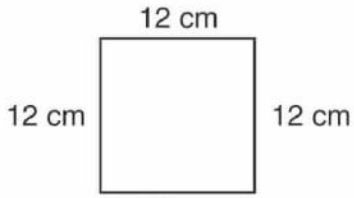


(iii)

Find the perimeter of each of these squares and note down your observations in the form of a table as given below:

Square	Side of the square (s)	Perimeter (P) = Sum of all sides
(i)	6 m	6 m + 6 m + 6 m + 6 m = 4 × 6 m
(ii)	8 cm	8 cm + 8 cm + 8 cm + 8 cm = 4 × 8 cm
(iii)	12 cm	12 cm + 12 cm + 12 cm + 12 cm = 4 × 12 cm

What do you conclude?

We conclude that the perimeter of a square = 4 × side of the square.

Thus, $P = 4 \times s$ or, $s = \frac{P}{4}$, where s is the side of the square.

Example 1 : Find the perimeter of a rectangle whose length and breadth are 15 cm and 7 cm respectively.

Solution : Length of the rectangle = 15 cm

Breadth of the rectangle = 7 cm

∴ Perimeter of the rectangle,

$$P = 2(l + b)$$
$$= 2(15 + 7)\text{ cm}$$
$$= 2 \times 22\text{ cm} = 44\text{ cm}$$

Thus, the perimeter of the rectangle is 44 cm.

Example 2 : The length of each side of a square is 14 m. Find its perimeter.

Solution : Side of the square = 14 m

∴ Perimeter of the square,

$$P = 4 \times s$$
$$= 4 \times 14\text{ m} = 56\text{ m}.$$

Example 3 : Find the length of a rectangle whose perimeter is 80 cm and breadth is 15 cm.

Solution : Perimeter of the rectangle = 80 cm

Breadth of the rectangle = 15 cm

$\therefore$ Length (l) of the rectangle $= \left(\frac{P}{2} - b\right)$

$= \left(\frac{80}{2} - 15\right)$ cm

$= (40 - 15)$ cm

$= 25$ cm.

Example 4 : Find the side of a square whose perimeter is 36 m.

Solution : Perimeter of the square = 36 m

$\therefore$ Side (s) of the square $= \frac{\text{Perimeter}}{4}$

$= \frac{36}{4}$ m

$= 9$ m.

Testing Time 11.2

1. Find the perimeter of the rectangle in which :

(a) Length = 9 cm, Breadth = 7 cm

(b) Length = 11.5 cm, Breadth = 9 cm

(c) Length = 12.5 m, Breadth = 7.3 m

(d) Length = 35.2 m, Breadth = 23 m

2. Find the perimeter of the square in which each side is as follows :

(a) 6 cm (b) 9.5 cm (c) 19.5 m (d) 25.4 dm

3. Find the breadth of the rectangle in which :

(a) Perimeter = 42 cm, Length = 12 cm

(b) Perimeter = 64 cm, Length = 19 cm

(c) Perimeter = 86 dm, Length = 29 dm

(d) Perimeter = 118 m, Length = 44 m

4. Find the each side of the square whose perimeter is as follows :

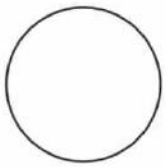
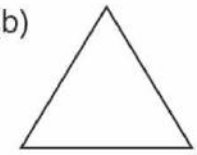
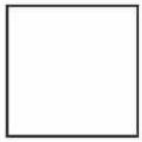
(a) 44 dm (b) 62 cm (c) 101 cm (d) 97 m

5. Find the perimeter of a rectangular park whose length and breadth are 52 m and 25 m respectively.
6. Find the perimeter of a square shaped plot whose each side is 36.2 dam.
7. The length of a rectangular field is 103 m and its perimeter is 352 m. What is the breadth of the field?
8. What is the length of each side of a carom-board if its perimeter is 322 cm?

AREA

CONCEPT OF AREA

If we draw a closed figure on a piece of paper and cut the paper along the boundaries of the figure, then the cutout gives the amount of surface enclosed by the figure.

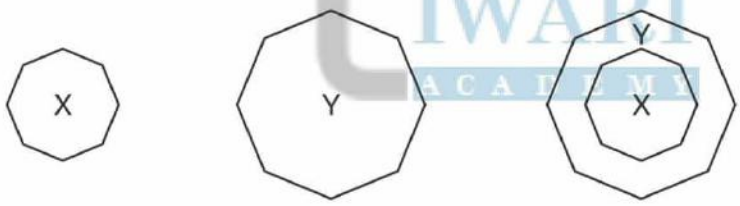
(a)  (b)  (c)  (d) 

The shaded regions shown above, show the amount of surfaces enclosed by a circle, a triangle, a rectangle and a square respectively.

The measure of the amount of surface enclosed by a figure is called its area.

Comparison of areas: When the given surfaces are of the same shape, we compare them by placing one over other.

Look at the two figures X and Y shown below.



By placing them one over the other, we find that Y is larger than X.

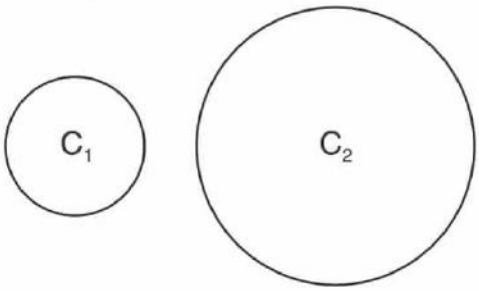
Thus, we can say that area of figure Y is larger than that of figure X.

Now, consider two circles C_1 and C_2 of radii 1 cm and 2 cm respectively.

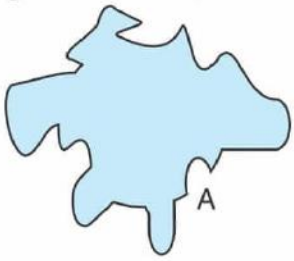
Obviously, the surface enclosed by the circle C_2 is more than that enclosed by circle C_1.

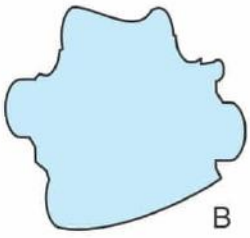
Thus, the area of circle C_2 is more than that of circle C_1.

Sometimes, it becomes difficult to compare the areas of the two surfaces by simply looking at them, particularly when the figures are of different shapes.



Look at the figures A and B, shown below.



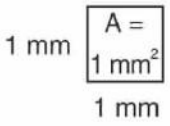


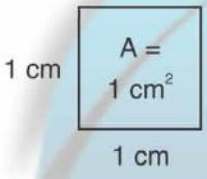
Obviously, we cannot compare the areas of figure A and figure B until and unless we measure them in definite units.

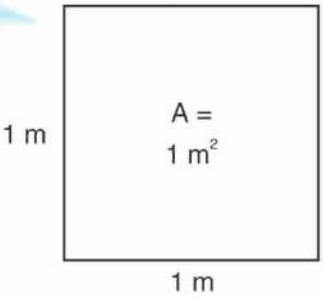
STANDARD UNITS OF AREA

The area of a small figure is measured in square millimetres or square centimetres. The bigger area is measured in square metres.

- The area of a square of side 1 mm is 1 square millimetre, written as 1 square mm or 1 mm^2.







- The area of a square of side 1 cm is 1 square centimetre, written as 1 square cm or 1 cm^2.
- The area of a square of side 1 metre is 1 square metre, written as 1 square m or 1 m^2.

AREA BY COUNTING SQUARES ON A SQUARE PAPER

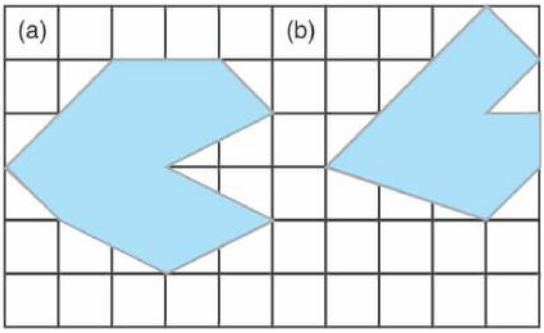
We can find area of some regular and irregular figures by using squared paper. The process has been explained through some examples.

Steps:

1. Count the number of complete squares enclosed by the figure.
2. Count the number of those squares in which more than half part is enclosed by the figure and treat them as one complete square.
3. Count the number of those squares in which half part is enclosed by the figure and divide it by 2 to get the number of complete squares.
4. Leave those squares in which less than half part is enclosed by the figure.

The sum of all squares obtained from steps 1 to 3 is the area of the figure.

Example 5 : Find the area of the following figures using square paper. Take area of each square = 1 sq. c.m.



Solution : (a) **Steps:**

1. Number of complete squares enclosed by the figure = 6
2. Number of more than half squares enclosed by the figure = 4
3. Number of half squares enclosed by the figure = 4

Hence, area of the figure $= (6 + 4 + \frac{1}{2} \times 4)$ sq. cm

$= (6 + 4 + 2)$ sq. cm $= 12$ sq. cm.

(b) Number of complete squares enclosed by the figure = 4

Number of more than half squares enclosed by the figure = 2

Number of half squares enclosed by the figure = 6

Hence, area $= (4 + 2 + \frac{1}{2} \times 6)$ sq. cm

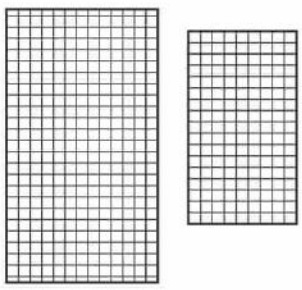
$= (4 + 2 + 3)$ sq. cm $= 9$ sq. cm.

TIWARI ACADEMY

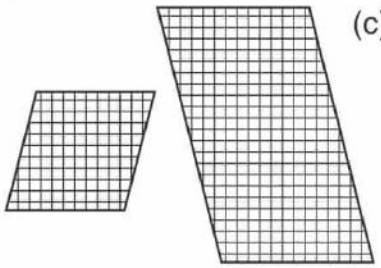
Testing Time 11.3

1. Which of the shaded regions has greater area?

(a)



(b)



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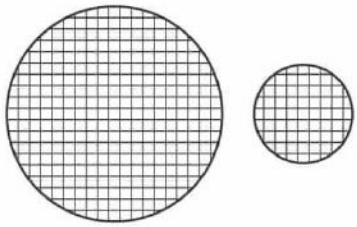
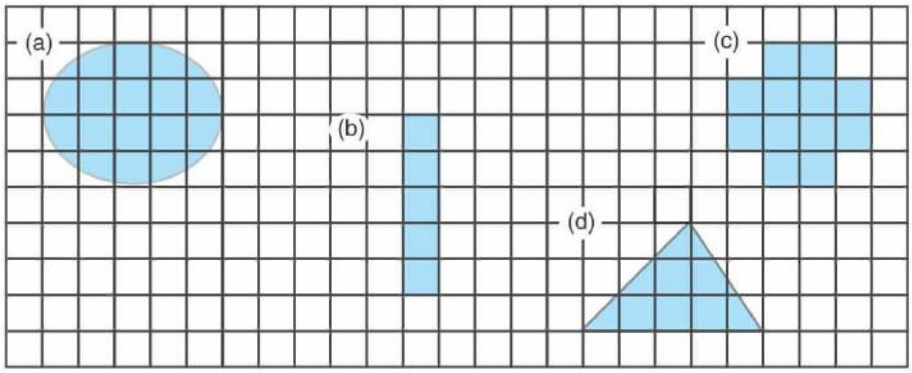
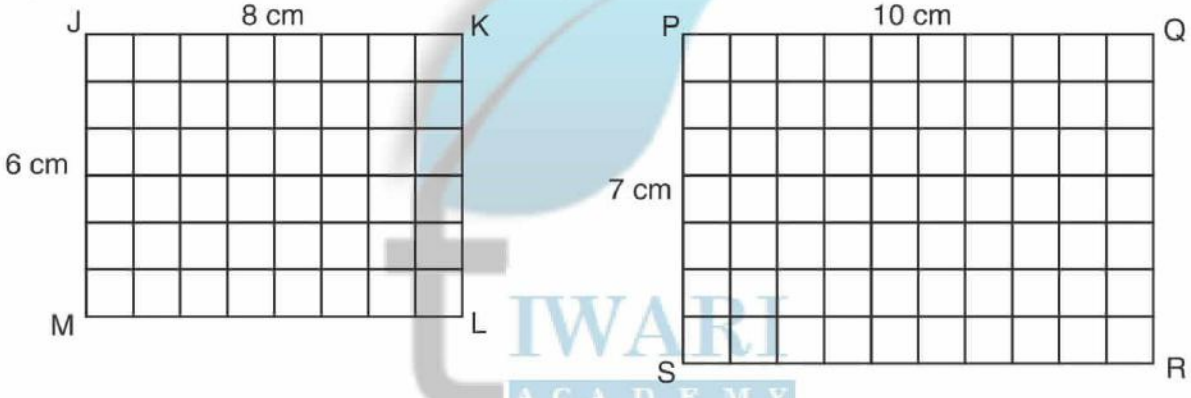


2. Find the areas of the figures drawn on a square paper by counting squares :



AREA OF A RECTANGLE AND A SQUARE

In the previous sections , we have learnt how to find the area of any given figure by counting the number of squares enclosed by the figure. This method is time consuming and a bit difficult for calculating the area of given figures. There is an easier method for calculating the areas of rectangles and squares. Consider the following examples.



Each of the above rectangles has been divided into squares of side 1 cm. The area of each square is 1 sq. cm. Find the area of the rectangle by counting the squares enclosed in each rectangle and record your observation as follows:

Area of rectangle JKLM = 48 sq. cm

Area of rectangle PQRS = 70 sq. cm

Now, look at the following table:

Rectangle	Length (l) in cm	Breadth (b) in cm	Area (A) in sq. cm
JKLM	8	6	48 or (8 × 6)
PQRS	10	7	70 or (10 × 7)

What do you observe?

We observe that the area of a rectangle (A) = length × breadth

or $\boxed{A = l \times b}$

In the case of a square, we know that l = b.

Hence, the area of a square (A) = side × side

or $\boxed{A = l \times l}$

Example 6 : Find the area of the rectangle whose length and breadth are 24 cm and 17 cm respectively.

Solution : Length of the rectangle = 24 cm

Breadth of the rectangle = 17 cm

Area of the rectangle = (24 × 17) sq. cm = 408 sq. cm.

Example 7 : Find the area of the square whose each side is 16 m.

Solution : Side of the square = 16 m

Hence, area of the square = (16 × 16) sq. m = 256 sq. m.

Testing Time 11.4

1. Find the area of the rectangle in which:

(a) l = 8 cm, b = 5 cm

(b) l = 11 dm, b = 7 dm

(c) l = 7.5 m, b = 6 m

(d) l = 12.3 m, b = 8.1 m

(e) l = 81 cm, b = 23 cm

(f) l = 29.1 cm, b = 12.16 cm

2. Find the area of the square whose each side is as follows:

(a) 8 dm

(b) 13 cm

(c) 26 cm

(d) 12.3 m

(e) 25.21 m

3. What is the area of a rectangular park whose length and breadth are 43 m and 29 m respectively ?

4. Find the area of a square field in which each side is 83 m.

5. The length and breadth of a rectangular field are 103 m and 67 m respectively. Find its area and also the cost of ploughing it at the rate of 25 paise per square meter.

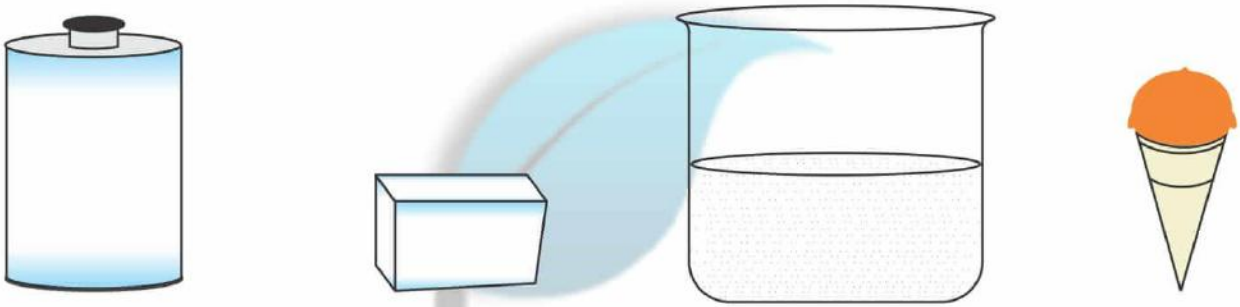
6. Find the length and area of a rectangle whose breadth and perimeter are 13 dm and 68 dm respectively.
7. Find the area of a chess-board whose perimeter is 92 cm.
8. Compare the areas of a rectangle of length 8.9 m and breadth 6.3 m and a square of side 7.2 m.

VOLUME

CONCEPT OF VOLUME

We are familiar with many solids / objects.

Look at the figures of some solids / objects given below:



(i) (ii) (iii) (iv)

Each of these objects occupies certain amount of space. The bigger objects occupy more space than the smaller objects.

Let us prove that every object occupies certain amount of space.

Take a beaker and pour some water in it [figure (i)]. Now, put a stone in the beaker such that it is completely immersed in the water [figure (ii)].

(i) (ii)

What do you observe?

We observe that the water level in the beaker has increased.

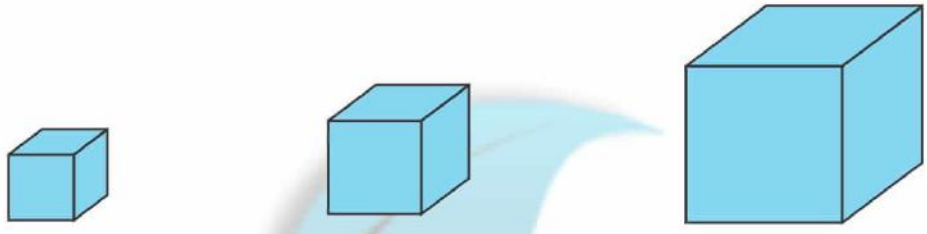
It shows that the stone has displaced some water which is equal to the amount of space occupied by it. It is because stone occupies certain space.

The amount or magnitude or measure of the space (region) occupied by a solid is called the volume of the solid.

STANDARD UNITS OF VOLUME

Small volumes are measured in cubic millimetres or cubic centimetres. Big volumes are measured in cubic metres.

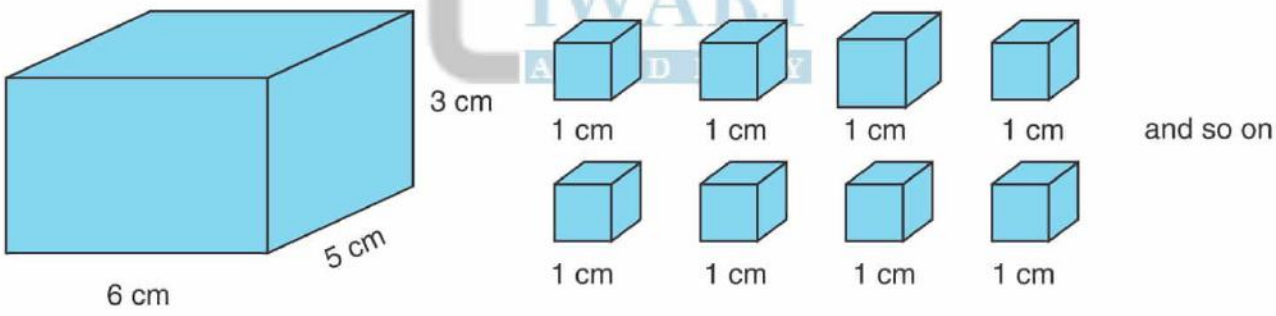
- The volume of a cube of edge 1 mm is 1 cubic millimetre, written as 1 cu. mm or 1 mm^3.
- The volume of a cube of edge 1 cm is 1 cubic centimetre, written as 1 cu. cm or 1 cm^3.
- The volume of a cube of edge 1 m is 1 cubic metre, writen as 1 cu. m or 1 m^3.



1 mm cube (i) 1 cm cube (ii) 1 m cube (iii)

FINDING VOLUME BY COUNTING CUBES

Activity 1: Take a cuboidal container of length say 6 cm, breadth 5 cm and height 3 cm and some cubes of side 1 cm as shown below:

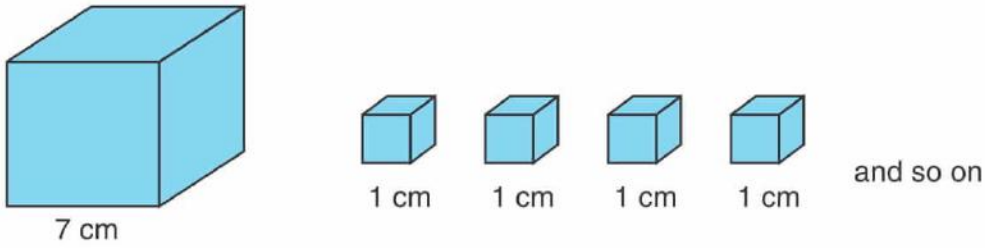


Now, fill up the container with these unit cubes, i.e., cubes of side 1 cm.

What do you observe?

We observe that 90 cubes of side 1 cm fill up the entire cuboidal container. Thus, the volume of the container is equal to 90 cu. cm.

Activity 2: Take a cubical container of side 7 cm and some cubes of side 1 cm as shown below:



Fill the cubical container with these small cubes.

What do you observe?

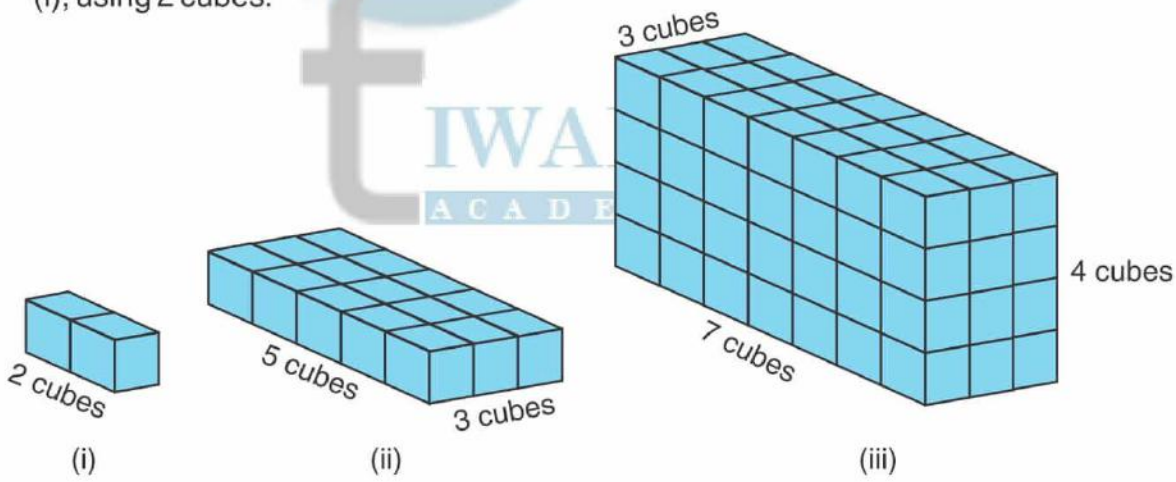
We observe that 343 smaller cubes fill up the entire bigger cube. Thus, the volume of the cubical container is equal to 343 cu. cm.

FORMULAE FOR VOLUME OF A CUBOID AND A CUBE

In the previous section, we have discussed the method of finding volume by counting the unit cubes. This method of finding volume is time consuming and also not practical in some cases such as for a cuboid of sides 6.2 cm, 4 cm and 2.5 cm.

Now, we shall obtain a formula for volume (V) of a cuboid by performing the following activity.

Activity 3: Take some unit cubes and place them in the form of a cuboid as shown in the figure (i), using 2 cubes.



Make another cuboid containing 3 rows of 5 unit cubes each as shown in figure (ii).

Again, make another cuboid of the type given in figure (iii).

Now, find the volume of each of these cuboids by counting cubes and record observations in the form of a table as given below:

Cuboid	Length (l) in cm	Breadth (b) in cm	Height (h) in cm	Volume in cu. cm
(i)	2	1	1	$2 = 2 \times 1 \times 1$
(ii)	5	3	1	$15 = 5 \times 3 \times 1$
(iii)	7	3	4	$84 = 7 \times 3 \times 4$

What do you conclude?

We conclude that the volume of a cuboid = length × breadth × height or, $V = l \times b \times h$

We know that in the case of a cube $l = b = h$.

Hence, the volume of a cube = side × side × side or, $V = l \times l \times l$

Testing Time 11.5

1. Find the volume of the cuboid in which :

(a) $l = 8\,cm, b = 4\,cm, h = 3\,cm$ (b) $l = 12\,cm, b = 9\,cm, h = 5\,cm$

(c) $l = 4\,m, b = 3.5\,m, h = 2\,m$ (d) $l = 14\,dm, b = 6.5\,dm, h = 3.5\,dm$

(e) $l = 7.2\,dm, b = 6.9\,dm, h = 3\,dm$ (f) $l = 26\,m, b = 19.5\,m, h = 13.2\,cm$

2. Find the volume of the cuboid whose each side is as follows:

(a) 4 dm (b) 7 cm (c) 12 m (d) 20 cm

(e) 6.3 cm (f) 5.1 m (g) 9.4 dm (h) 7.5 m

3. What is the volume of cuboid if its length, breadth and height are 10 cm, 8.5 cm and 3.5 cm respectively?
4. What is the volume of a cube of side 8.7 m ?
5. Find the volume of water in a cuboidal vessel of 13 dm length, 9.5 dm breadth and 6.2 dm height.
6. The length, breadth and height of a brick are 21 cm, 10 cm and 6.5 cm respectively. Find the space occupied by such 400 bricks.
7. From a 15 cm × 10 cm × 5 cm cuboid, how many cubes of edge 5 cm can be formed ?
8. Compare the volumes of a cuboid of length 13 cm, breadth 9.5 cm and height 6 cm and a cube of side 9.2 cm.