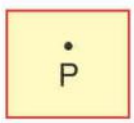
Chapter 2
Geometry: Angle

In the earlier classes, we have learnt about some basic concepts of geometry such as point, line, line segment and ray. Now, we shall study about them in detail. We shall also learn about the concepts of a plane, an angle, measuring an angle and various kinds of angles.

POINT

A dot (.) represents a point. It has no length, breadth or height. A point is named or labeled with the help of a capital letter of English alphabet. A dot shown on the right side represents a point P.



LINE SEGMENT

A line segment is the shortest distance or straight path between any two points. The figure shown below is a line segment PQ.

It is symbolically written as $\overline{PQ}$ and read as line segment PQ. P and Q are called the end-points of the line segment PQ.

Thus, a line segment has two end points and definite length. So, length of a line segment can be measured.

LINE

A line segment extended endlessly on both sides is called a line.

If a line segment AB is extended on both sides and marked by arrows at the end points then we find a line AB.

The line AB is symbolically written as $\overleftrightarrow{AB}$. Arrow-cap over both ends A and B shows that it is extended infinitely in both directions. Since a line does not have any end-point, we cannot draw it on paper.

RAY

A line segment extended endlessly in one direction is called a ray.

Thus, if we extend a line segment AB in one direction say from A to B then we find a ray AB. The ray AB is denoted as $\overrightarrow{AB}$.

Clearly, a ray AB has one end point, namely A.

This end point is called the initial point of $\overrightarrow{AB}$.

Since, a ray is endless in one direction, it cannot be drawn on a paper.

So, the length of a ray cannot be measured. We can simply represent it as shown above.

Note : $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are two different rays. $\overrightarrow{AB}$ is extended from A to B whereas $\overrightarrow{BA}$ is extended from B to A.

PLANE

7 + 2 = 9

Look at the picture of a black-board shown on the right. Its surface is flat.

If this flat surface is extended infinitely in all directions then it is called a plane.

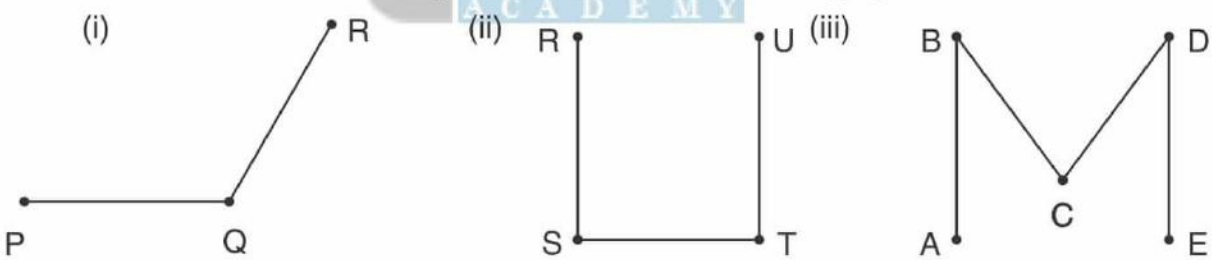
Thus, a plane is a flat surface extended infinitely in all directions.

A plane has no boundary, i.e., we cannot draw a plane on a paper. We can only draw a part of a plane.

The surface of walls of a room, book, slates etc. are examples of a plane.

Now, study some examples to understand the concepts discussed so far.

Example 1 : Find the number of line segments in each of the following figures :



Solution : In figure (i), there are two line segments, namely PQ and QR.

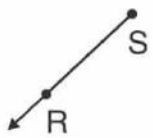
In figure (ii), there are three line segments, namely RS, ST and TU.

In figure (iii), there are four line segments, namely AB, BC, CD and DE.

Example 2 : Write the names of rays in the following figures :

(i) 

(ii) 

(iii) 

(iv) 

Solution : We name the given rays as:

In figure (i) ray PQ or $\overrightarrow{PQ}$

In figure (ii) ray MN or $\overrightarrow{MN}$

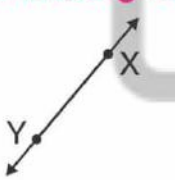
In figure (iii) ray SR or $\overrightarrow{SR}$

In figure (iv) ray XY or $\overrightarrow{XY}$.

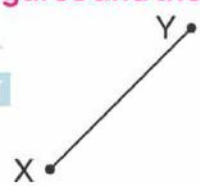
Testing Time 2.1

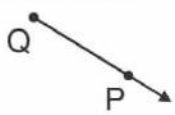
1. Which one has a fixed length – a line segment or a line?
2. Does a plane have any boundary ?
3. **Fill in the blanks:**
   (a) A line has _____ end points.
   (b) A line segment has _____ end points.
   (c) A line segment extended endlessly in one direction is called a _____ .
4. **Identify the rays, line or line segments in the following figures and then name them:**

(a) 

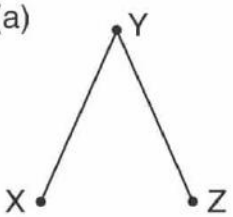
(b) 

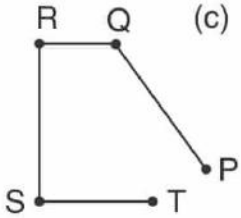
(c) 

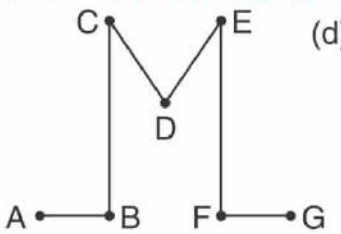
(d) 

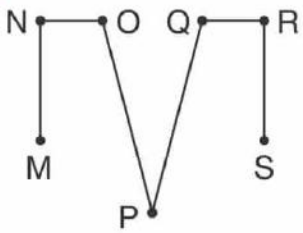
(e) 

5. **How many line segments are there in each of the following figures?**

(a) 

(b) 

(c) 

(d) 

6. **Draw:** (a) a line MN (b) a ray AB (c) a line segment PQ

ANGLES

There are many physical objects which have two arms joined together by a hinge. For example, two arms of a divider, two hands of a clock, two sharp parts of scissors are hinged at a point such that they are inclined towards each other and have an opening between them. These objects give us an idea of an angle in geometry as discussed below.

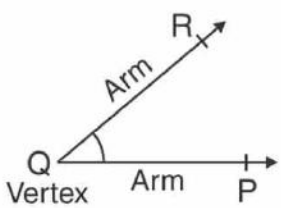
Divider

Clock

Scissors

ANGLE

A figure formed by two rays with same initial point is called an angle.



The rays forming an angle are called its arms or sides and the common initial point is called the vertex of the angle.

In the figure given here, rays QP and QR are the arms and Q is the vertex of the angle.

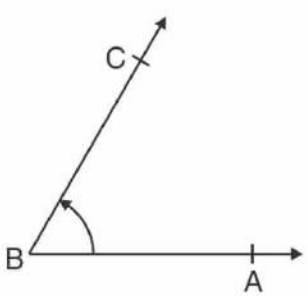
The arms are often joined by a small circular arc near the vertex as shown in the figure given above.

NAMING AN ANGLE

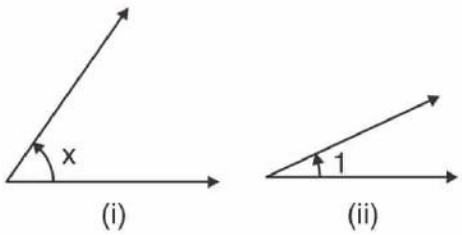
In order to identify angles, when there are more than one in the same plane, we name the angles. There are several ways of naming angles:

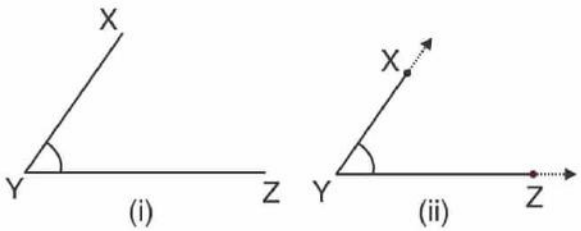
In order to name a given angle, formed by two rays with same initial point, we name the vertex and name one more point on each arm.

In the figure shown on the right, the angle can be named as ∠ABC or ∠CBA and read as 'angle ABC' or 'angle CBA'. The symbol ' ∠' stands for the angle. It should be noted that in either case the letter B denoting the vertex is written in the middle and the two extreme letters are any points on the arms of the angle.



- Sometimes, we name an angle by writing the letter at the vertex of the angle. For example, ∠ABC can also be written as ∠B or angle B.
- An angle is also named by placing a small letter x, y, z etc. {figure (i)} or a number 1, 2, 3 etc. {figure (ii)} near the small circular arc connecting the two arms of the angle.

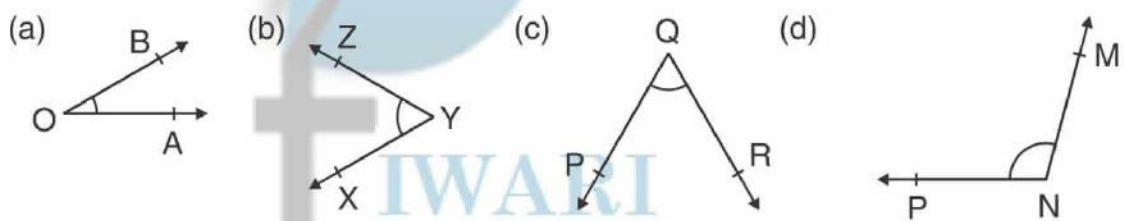




Note : Quite often, we come across two line segments XY and YZ with a common end point Y as shown in figure (i). We then say that an angle has been determined at Y by line segments XY and YZ. Thus, two line segments with a common end point also determine an angle at that point.

By this statement we mean that angle is formed by the rays, corresponding to these line segments in figure (ii). However, we shall not give much stress on rays and line segments in connection with an angle.

Example 3 : Look at the following angles and write their names :



Solution : The names of the angles are:

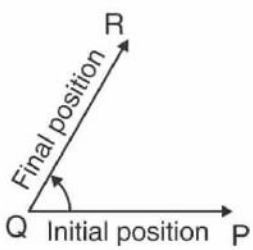
(a) ∠AOB or ∠BOA (b) ∠XYZ or ∠ZYX

(c) ∠PQR or ∠RQP (d) ∠MNP or ∠PNM

ANGLE AS ROTATION OF A RAY

An angle can also be described by rotating a ray as discussed below.

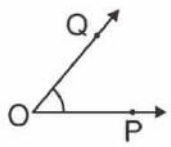
Let there be a ray with initial point Q. Suppose it starts rotating in the plane from the initial position QP about the point Q until it reaches the final position QR as shown in the figure. Then, we say that an angle PQR has been described by the rotating ray QP with Q as vertex and its arms as QP and QR.

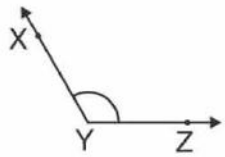


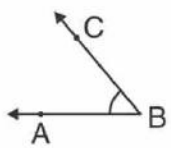
The magnitude of an angle is the amount of rotation through which one of the arms must be rotated about the vertex to bring it to the position of the other arm.

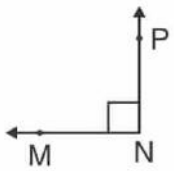
MEASURING AN ANGLE

Look at the following pairs of angles:

(i) 

(ii) 

(iii) 

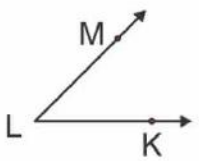
(iv) 

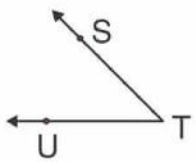
What do you observe?

We observe that in figure (i) and (ii) inclination or opening between two rays OP and OQ is less than the inclination or opening between two rays YX and YZ. So, we say that $\angle POQ$ is smaller than $\angle XYZ$.

Similarly, in figure (iii) and (iv), $\angle ABC$ is less than $\angle MNP$.

Sometimes, it is easy to decide that which angle is greater or smaller than the other after looking at the figure. However, in some cases it is not easy to decide that which angle is greater (or smaller) as in the case of the following figures:

(i) 

(ii) 

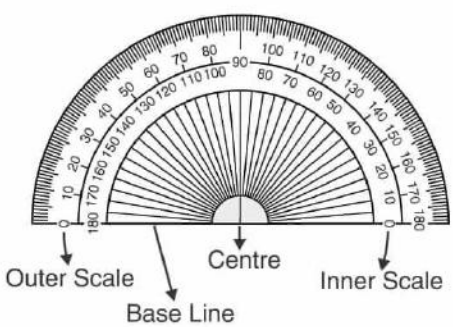
To overcome this difficulty, we need to measure an angle exactly.

The angle is measured in degrees.

A complete turn around a point is divided into 360 equal parts. Each of these parts measures 1 degree and it is written as 1°.

Thus, a complete turn around a point measures 360°.

To measure the number of degrees in an angle, we make use of an instrument called Protractor. It is a semi-circular piece of plastic or metal. There are two semi-circular scales on it. One of them contains 0° to 180° written in anticlockwise direction and is called an inner scale. The other contains 0° to 180° written in clockwise direction and is called an outer scale. Each sub-division represents 1°.

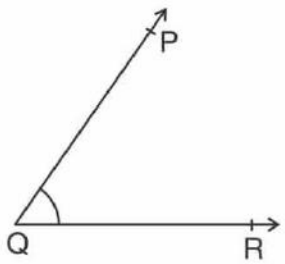


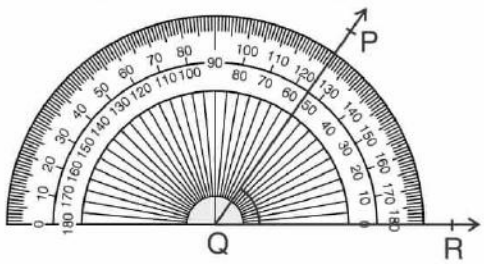
We can read a protractor from left to right or from right to left. The line segment joining 0° and 180° marks is called the base line of the protractor. The mid-point of the base line is called the centre of the protractor.

Let us measure ∠PQR (see figure given below) using a protractor.

Method: Place the protractor in such a way that its centre is exactly on the vertex Q of the angle and the base line lies along the arm QR.

Now, note the mark on the rim of the protractor through which arm QP passes.





Taking the scale whose 0° mark lies on QR, we find that OB passes through 55° mark.

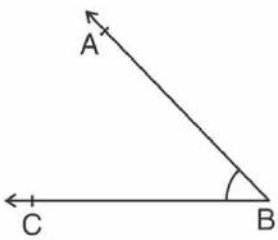
Hence, m ∠PQR = 55°.

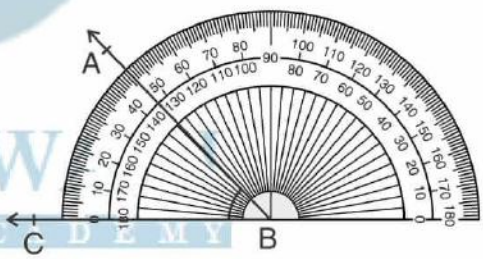
Note that here we have read the markings of the inner scale of the protractor.

Now, let us measure ∠ABC, again by using the same procedure as mentioned above.

For this, again we place the protractor in such a way that its centre is exactly on the vertex B of the angle and the base line lies along the arm BC.

Now, note the mark on the rim of the protractor through which arm BA passes.



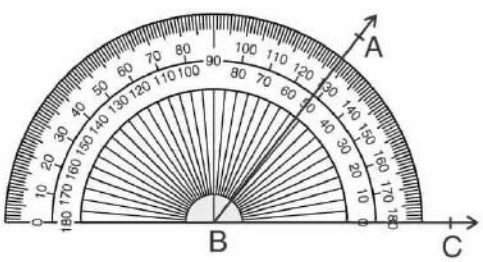


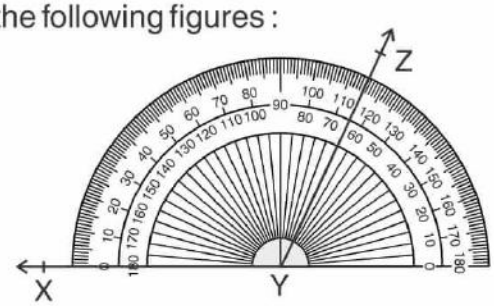
Taking the scale whose 0° mark lies on BC, we find that BA passes through 46°.

Hence, m ∠ABC = 46°.

Note that here for ∠ABC, we read the markings of outer scale.

Example 4 : Read the measures of ∠ABC and ∠XYZ in the following figures :





Solution : We find that, ∠ABC = 52° and ∠XYZ = 115°.

CONSTRUCTION OF ANGLES

We use a protractor to measure angles. We also use it to draw angles of given measurements. Study the following examples.

Example 5 : Draw an angle of 50°.

Solution : To draw an angle of 50°, we take the following steps:

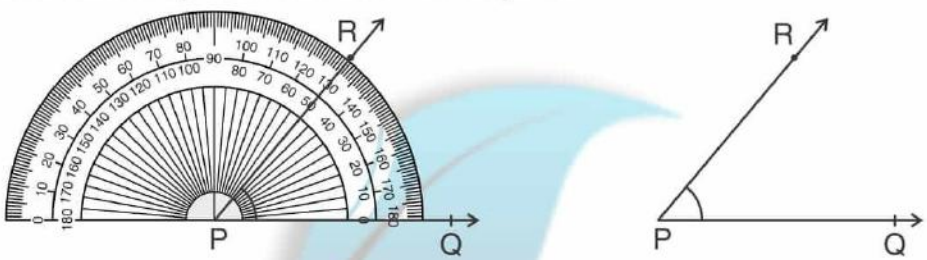
Step 1. Draw a ray, say PQ, with the end point P on paper.

Step 2. Place the protractor in such a way that its centre point lies on P and its base line lies along PQ.

Step 3. Run your eyes along the scale whose 0° mark lies on PQ until you find the 50° mark on the rim.

Step 4. On that mark put a dot with a fine pencil and name it R.

Step 5. Remove the protractor and draw a ray PR.



Then, ∠RPQ is the required angle such that m ∠RPQ = 50°.

Example 6 : Draw an angle of 125°.

Solution : To draw an angle of 125°, we take the following steps:

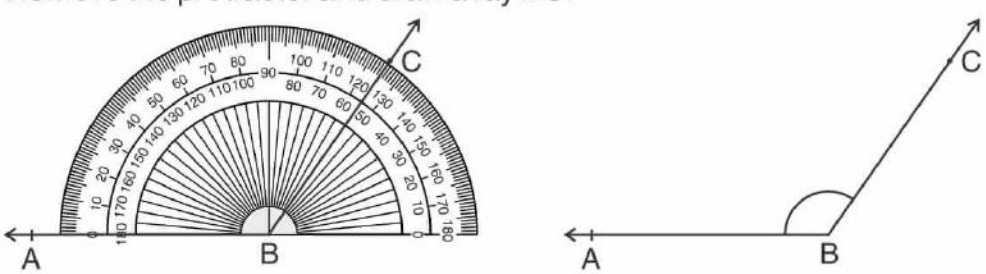
Step 1. Draw a ray, say AB with the end point B on paper.

Step 2. Place the protractor in such a way that its centre point lies on B and its base line lies along BA.

Step 3. Run your eyes along the scale whose 180° mark lies on BA until you find 125° mark on the rim.

Step 4. On that mark put a dot with a fine pencil and name it C.

Step 5. Remove the protractor and draw a ray BC.

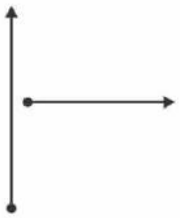


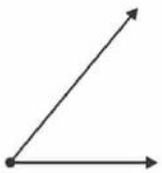
Then, ∠ABC is the required angle such that m ∠ABC = 125°.

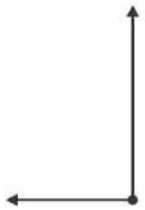
Testing Time 2.2

1. Which of the following figures represent angle?

(a) 

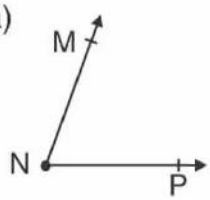
(b) 

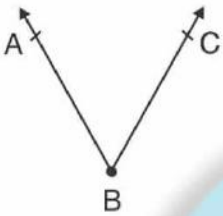
(c) 

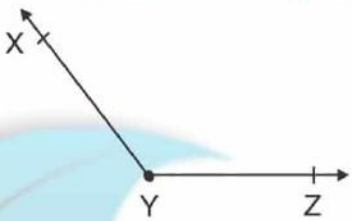
(d) 

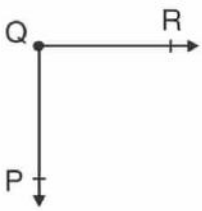
(e) 

2. Write the names of vertex and the arms of each of the following angles:

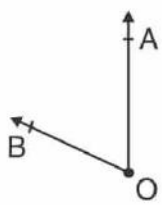
(a) 

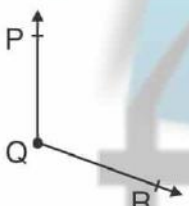
(b) 

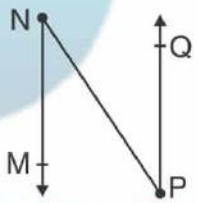
(c) 

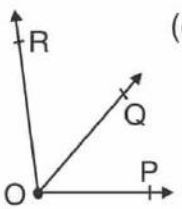
(d) 

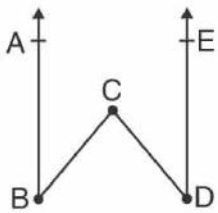
3. Write the name of the angle/ angles in each of the following figures:

(a) 

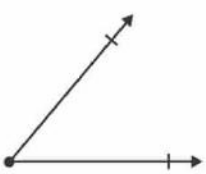
(b) 

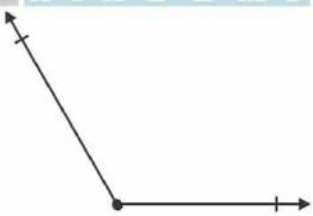
(c) 

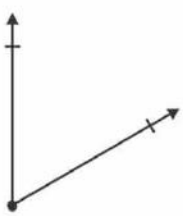
(d) 

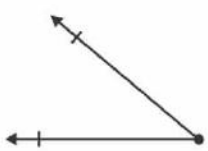
(e) 

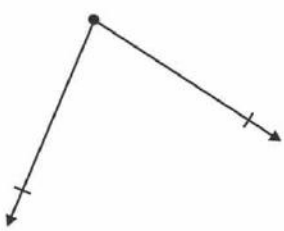
4. Measure the angle in each of the following figures with the help of a protractor and write the measure in degrees:

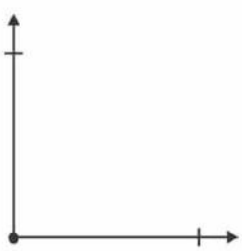
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

5. Construct angles of the following measures with the help of protractor and ruler:

(a) 60° (b) 90° (c) 25°

(d) 73° (e) 110° (f) 165°

6. Construct an angle POQ of measure 60° in which the base OP is 5 cm.

CLASSIFICATION OF ANGLES

On the basis of magnitude, the angles are classified as follows:

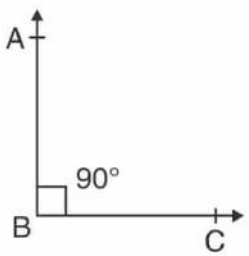
Zero Angle

An angle whose measure is 0° is called a zero angle.

In the given figure, ∠POP is a zero angle.

Right Angle

An angle whose measure is 90° is called a right angle.



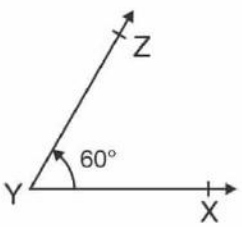
In the given figure, ∠ABC = 90°.

Hence, it is a right angle.

We indicate an angle of 90° by the symbol as shown in the figure.

Acute Angle

An angle whose measure lies between 0° and 90° is called an acute angle.

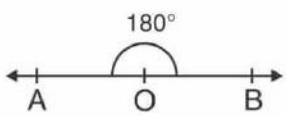


In other words, an angle which is greater than a zero angle but less than a right angle is called an acute angle.

In the given figure, ∠XYZ = 60°. Hence, it is an acute angle.

Straight Angle

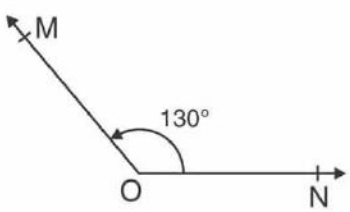
An angle whose measure is 180° is called a straight angle.



In the given figure, ∠AOB = 180°. Hence, it is a straight angle.

Obtuse Angle

An angle whose measure lies between 90° and 180° is called an obtuse angle.



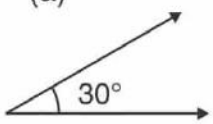
In other words, an angle which is greater than a right angle but less than a straight angle is called an obtuse angle.

In the given figure, ∠MON = 130°. Hence, it is an obtuse angle.

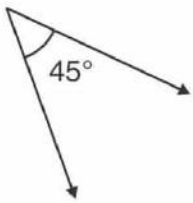
Testing Time 2.3

1. Classify each of the following figures into acute, right, obtuse or straight angles:

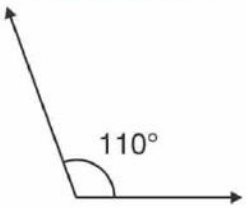
(a)



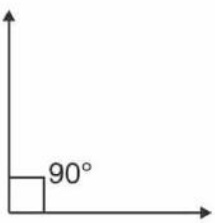
(b)



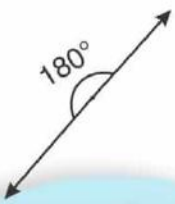
(c)



(d)



(e)



2. Fill in the blanks:

(a) An angle which measures between 90° and 180° is called an ..... angle.

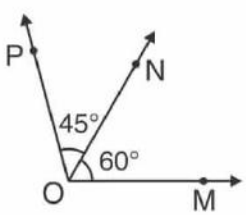
(b) An angle of 90° is called a ..... angle.

(c) An angle which measures less than 90° is called an ..... angle.

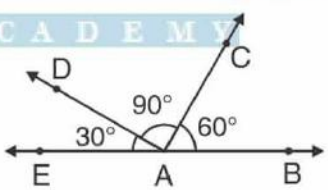
(d) An angle of 0° is called a ..... angle.

3. Write the names of acute angles in each of the following figures:

(a)



(b)



(c)