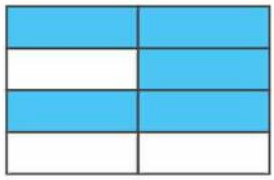
Chapter 4
Fractions

FRACTIONAL PART OF A COLLECTION

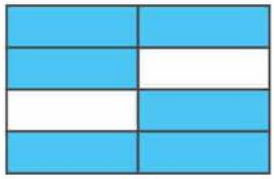
In the previous classes we have learnt the meaning of a collection in the following two ways :

1. Part of a whole



A fractional number or a fraction indicates a part or parts of a whole.

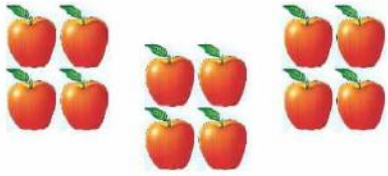
For example, the coloured part of the figure represents $\frac{5}{8}$ of the whole, i.e., $\frac{5}{8}$ indicates 5 parts out of 8 equal parts of a whole.



Similarly, $\frac{6}{8}$ indicates 6 parts out of 8 equal parts of a whole.

2. Part of a collection or group of objects

To understand fractional part of a collection, let us consider a collection of 12 apples.

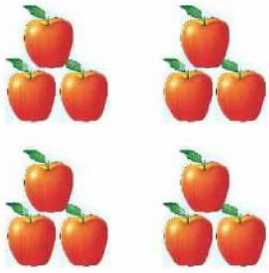


If we divide the collection into three equal parts, we get 4 apples in each of the three parts.

Thus, one-third of 12 is 4, i.e.,

$$\frac{1}{3} \text{ of } 12 = 12 \times \frac{1}{3} \text{ or } 12 \div 3 = 4.$$

Similarly, to obtain $\frac{3}{4}$ of the collection, we divide the collection into four equal parts and take 3 such parts. In the given figure, there are 12 apples in 3 parts out of 4 equal parts of the collection, i.e.,

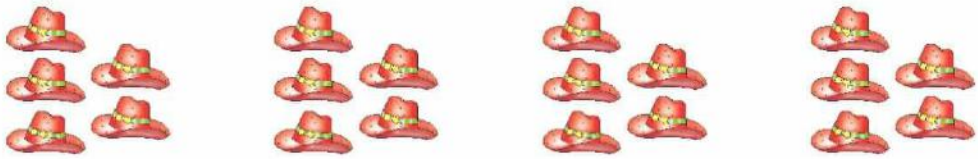


$$\frac{3}{4} \text{ of } 12 = 24 \times \frac{3}{4} \text{ or } (12 \times 3) \div 4 = 9$$

Therefore, we conclude that the fractional part of a collection is determined by dividing the collection into subgroups equal to the number representing the denominator of the fraction. We then take the number of subgroups equal to the number representing the numerator of the fraction.

Testing Time 4.1

1. Look at the figure given below and answer the questions that follow:



(a) How many hats are there in all?

(b) Find $\frac{1}{2}$ of the total number of buttons.

(c) Find $\frac{4}{5}$ of the total number of buttons.

2. What fractional part of the collection of 24 cups do the following crossed cups represent ?

(a)

(b)

3. Find $\frac{2}{4}$ of a collection of:

(a) 8 stars (b) 12 tables (c) 18 apples

(d) 42 dogs (e) 56 pencils (f) 72 pens

4. Find $\frac{2}{7}$ of a collection of:

(a) 14 mangoes (b) 28 toffees (c) 35 pens

(d) 49 toys (e) 63 birds (f) 77 kites

5. Find $\frac{3}{5}$ of a collection of:

(a) 10 balls (b) 15 dolls (c) 25 balloons

(d) 45 coins (e) 50 birds (f) 70 marbles

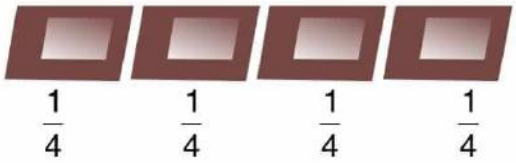
FRACTION AS DIVISION

If 4 chocolates are distributed equally among 4 students, how many chocolates does a student get?

Clearly, $4 \div 4 = 1$.

Similarly, if 12 chocolates are distributed equally among 4 students then each student will get $12 \div 4 = 3$ chocolates.

But if 1 chocolate is to be distributed among 4 students, then how many chocolates will a student get?



In this case also a student get $1 \div 4$, i.e., $\frac{1}{4}$.

Similarly, if 5 chocolates are to be divided equally among 4 students, then each will get $5 \div 4 = \frac{5}{4}$.

What do you observe?

We observe that a fraction can be expressed as a division. Conversely, division can be expressed as a fraction.

Testing Time 4.2

1. Write each of the following as a fraction:

(a) $8 \div 2$ (b) $12 \div 3$ (c) $45 \div 5$ (d) $48 \div 8$

(e) $63 \div 7$ (f) $78 \div 13$ (g) $84 \div 21$ (h) $150 \div 25$

2. Write each of the following as a division:

(a) $\frac{3}{5}$ (b) $\frac{6}{11}$ (c) $\frac{9}{13}$ (d) $\frac{7}{22}$

(e) $\frac{10}{19}$ (f) $\frac{18}{29}$ (g) $\frac{25}{48}$ (h) $\frac{32}{105}$

KINDS OF FRACTIONS

Like Fractions: Fractions having the same denominators are called like fractions.

For example, $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ etc. are like fractions.

Unlike Fractions: Fractions having different denominators are called unlike fractions.

For example, $\frac{2}{3}, \frac{5}{6}, \frac{7}{8}, \frac{8}{9}$ etc. are unlike fractions.

Unit Fractions: A fraction having numerator 1 is called a unit fraction.

For example, $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}$ etc. are unit fractions.

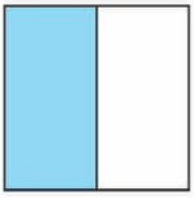
Proper Fractions: A fraction in which the numerator is less than the denominator is called a proper fraction.

For example, $\frac{3}{4}, \frac{7}{9}, \frac{13}{16}, \frac{25}{28}$ etc. are proper fractions.

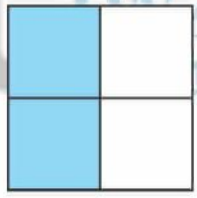
Improper Fractions: A fraction in which the numerator is greater than the denominator is called an improper fraction.

For example, $\frac{8}{5}, \frac{9}{4}, \frac{15}{7}, \frac{28}{16}$ etc. are improper fractions.

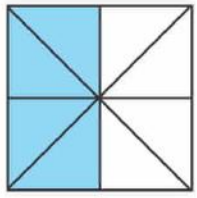
Equivalent Fractions: Two or more fractions which indicate the same fractional number are called equivalent fractions.



$\frac{1}{2}$



$\frac{2}{4}$



$\frac{4}{8}$

It is clear from the above figure that $\frac{1}{2}, \frac{2}{4}$ and $\frac{4}{8}$ are equivalent fractions.

We can express the above fractions as

$$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4} \qquad \frac{1}{2} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8}$$

What do you conclude ?

We conclude that an equivalent fraction of a given fraction can be obtained by multiplying the numerator and denominator by the same number (except zero).

Further, $\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$

$\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$

What do you conclude?

We conclude that an equivalent fraction of a given fraction can be obtained by dividing its numerator and denominator by their common factor (other than 1), if any.

In view of the above, we can generate equivalent fractions of any given fraction by:

- multiplying its numerator and denominator by the same number (other than 0) or,
- dividing its numerator and denominator by their common factor (other than 1), if any.

These are illustrated below:

$\frac{3}{7} = \frac{3 \times 2}{7 \times 2} = \frac{6}{14}$

$\frac{3}{7} = \frac{3 \times 3}{7 \times 3} = \frac{9}{21}$

Thus, $\frac{3}{7}$, $\frac{6}{14}$ and $\frac{9}{21}$ are equivalent fractions.

Similarly, $\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$

$\frac{21}{28} = \frac{21 \div 7}{28 \div 7} = \frac{3}{4}$

Thus, $\frac{12}{16}$, $\frac{21}{28}$ and $\frac{3}{4}$ are equivalent fractions.

Example 1 : Replace $\square$ by the correct numeral :

(a) $\frac{2}{5} = \frac{8}{\square}$ (b) $\frac{9}{15} = \frac{\square}{5}$

Solution : (a) We look at the numerators.

Since, $8 = 2 \times 4$, so we multiply both the numerator and the denominator of $\frac{2}{5}$ by 4.

$$\frac{2}{5} = \frac{2 \times 4}{5 \times 4} = \frac{8}{20}$$

Thus, replacing $\square$ by 20, we get $\frac{2}{5} = \frac{8}{20}$.

(b) We look at the denominators.

Since, $5 = 15 \div 3$, so we divide both the numerator and the denominator of $\frac{9}{15}$ by 3.

$$\frac{9}{15} = \frac{9 \div 3}{15 \div 3} = \frac{3}{5}$$

Thus, replacing $\square$ by 3, we get $\frac{9}{15} = \frac{3}{5}$.

Example 2 : Find an equivalent fraction of $\frac{8}{15}$ whose numerator is 32.

Solution : Since, $32 = 8 \times 4$, so we multiply both the numerator and the denominator of $\frac{8}{15}$ by 4.

$$\frac{8}{15} = \frac{8 \times 4}{15 \times 4} = \frac{32}{60}$$

Hence, $\frac{32}{60}$ is the required fraction.

Example 3 : Find an equivalent fraction of $\frac{4}{9}$ whose denominator is 63.

Solution : Since, $63 = 9 \times 7$, so we multiply both the numerator and the denominator of $\frac{4}{9}$ by 7.

$$\frac{4}{9} = \frac{4 \times 7}{9 \times 7} = \frac{28}{63}$$

Hence, $\frac{28}{63}$ is the required fraction.

We can check whether the two fractions are equivalent or not by cross multiplication.

If two fractions are equivalent, then

> Numerator of the first × Denominator of the second =
> Denominator of the first × Numerator of the second.

Example 4 : Check whether the given fractions are equivalent :

(a) $\frac{5}{9}, \frac{10}{18}$ (b) $\frac{6}{11}, \frac{30}{44}$

Solution : (a) By cross multiplication, we have

$5 \times 18 = 90$ and $9 \times 10 = 90$

Since the two products are the same, the given fractions are equivalent.

(b) By cross multiplication, we have

$6 \times 44 = 264$ and $11 \times 30 = 330$

Since the two products are not the same, the given fractions are not equivalent.

MIXED FRACTIONS OR MIXED NUMERALS

A whole number together with a proper fraction is called a mixed fraction or a mixed numeral.

For example, numbers such as $1\frac{2}{5}$ and $2\frac{3}{5}$ are mixed fractions or mixed numerals.

In $1\frac{2}{5}$ and $2\frac{3}{5}$, 1 and 2 are whole numbers and $\frac{2}{5}$ and $\frac{3}{5}$ are proper fractions.

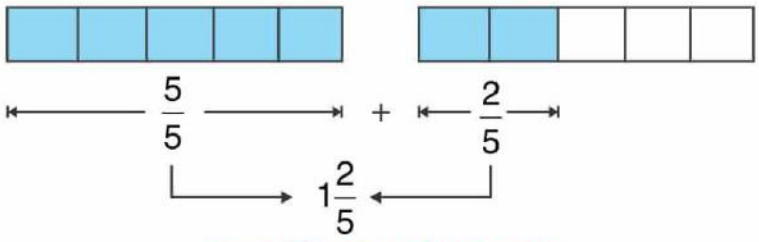
CONVERSION OF IMPROPER FRACTIONS INTO MIXED NUMBERS (MIXED FRACTIONS) AND VICE-VERSA

Let us consider some improper fractions:

$$\frac{7}{5} = \frac{5+2}{5} = \frac{5}{5} + \frac{2}{5} = 1 + \frac{2}{5}, \text{ or } 1\frac{2}{5}$$

> Mixed number
> $= \text{quotient} + \frac{\text{remainder}}{\text{divisor}}$

Pictorially, it can be shown as



Similarly, $\frac{13}{8} = \frac{8+5}{8} = \frac{8}{8} + \frac{5}{8} = 1 + \frac{5}{8}$, or $1\frac{5}{8}$

$\frac{22}{9} = \frac{18+4}{9} = \frac{18}{9} + \frac{4}{9} = 2 + \frac{4}{9}$, or $2\frac{4}{9}$

So, $1\frac{2}{5}$, $1\frac{5}{8}$ and $2\frac{4}{9}$ are mixed numbers.

We have converted above improper fractions into mixed numbers. Conversely, we can convert a mixed number into an improper fraction.

Let us consider the mixed numbers $1\frac{4}{7}$ and $3\frac{5}{12}$.

$$1\frac{4}{7} = 1 + \frac{4}{7} = \frac{1 \times 7}{7} + \frac{4}{7} = \frac{7+4}{7} = \frac{11}{7}$$

Similarly, $3\frac{5}{12} = 3 + \frac{5}{12} = \frac{3 \times 12}{12} + \frac{5}{12} = \frac{36+5}{12} = \frac{41}{12}$

A simple and quick way to convert a mixed number into an improper fraction is given below:

$$1\frac{4}{7} = \frac{1 \times 7 + 4}{7} = \frac{11}{7}; 3\frac{5}{12} = \frac{3 \times 12 + 5}{12} = \frac{41}{12}.$$

Testing Time 4.3

1. Which of the following are groups of like fractions?

(a) $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{5}{7}, \frac{6}{7}$ (b) $\frac{1}{11}, \frac{7}{11}, \frac{4}{7}, \frac{8}{13}, \frac{13}{6}$ (c) $\frac{2}{8}, \frac{5}{15}, \frac{13}{9}, \frac{6}{7}, \frac{4}{15}$

(d) $\frac{4}{13}, \frac{2}{13}, \frac{7}{13}, \frac{5}{13}, \frac{8}{13}$ (e) $\frac{3}{8}, \frac{5}{8}, \frac{1}{8}, \frac{7}{8}, \frac{9}{8}$ (f) $\frac{4}{9}, \frac{3}{10}, \frac{5}{8}, \frac{3}{5}, \frac{7}{10}$

2. Which of the following fractions are proper fractions?

(a) $\frac{5}{8}$ (b) $\frac{12}{17}$ (c) $\frac{13}{8}$ (d) $\frac{24}{27}$ (e) $\frac{42}{38}$

3. Which of the following fractions are improper fractions?

(a) $\frac{9}{10}$ (b) $\frac{22}{14}$ (c) $\frac{2}{3}$ (d) $\frac{16}{20}$ (e) $\frac{43}{38}$

4. Which of the following are unit fractions?

(a) $\frac{6}{9}$ (b) $\frac{2}{5}$ (c) $\frac{1}{6}$ (d) $\frac{11}{15}$ (e) $\frac{1}{8}$

5. Check whether the given fractions are equivalent :

(a) $\frac{2}{5}, \frac{8}{20}$ (b) $\frac{5}{9}, \frac{36}{42}$ (c) $\frac{3}{7}, \frac{9}{21}$

(d) $\frac{4}{11}, \frac{24}{55}$ (e) $\frac{6}{13}, \frac{30}{65}$ (f) $\frac{8}{15}, \frac{60}{32}$

6. Replace $\square$ in each of the following by the correct number :

(a) $\frac{5}{6} = \frac{10}{\square}$ (b) $\frac{4}{13} = \frac{12}{\square}$ (c) $\frac{8}{11} = \frac{\square}{44}$

(d) $\frac{9}{15} = \frac{\square}{60}$ (e) $\frac{1}{5} = \frac{\square}{15}$ (f) $\frac{3}{8} = \frac{24}{\square}$

7. Find the equivalent fraction of :

(a) $\frac{4}{7}$ having numerator 12 (b) $\frac{3}{8}$ having denominator 32

(c) $\frac{9}{11}$ having denominator 99 (d) $\frac{8}{14}$ having numerator 4

(e) $\frac{20}{60}$ having numerator 2 (f) $\frac{20}{35}$ having denominator 7

FRACTIONS IN LOWEST TERMS

A fraction is said to be in its lowest term (or in simplest form), if the HCF of numerator and denominator is 1.

Fractions $\frac{1}{2}, \frac{2}{3}, \frac{5}{7}, \frac{9}{13}$ are all in their lowest terms. To express a fraction in its lowest term, we divide the numerator and denominator by their HCF.

Example 5 : Reduce these fractions in their lowest terms:

(a) $\frac{8}{20}$ (b) $\frac{21}{54}$

Solution : (a) Here, numerator is 8 and denominator is 20.
Let us find the HCF of 8 and 20.
Factors of 8 are 1, 2, 4 and 8.
Factors of 20 are 1, 2, 4, 5, 10 and 20.

Common factors of 8 and 20 are 1, 2 and 4.

$\therefore$ HCF of 8 and 20 is 4.

Thus, $\frac{8}{20} = \frac{8 \div 4}{20 \div 4} = \frac{2}{5}$

Thus, $\frac{2}{5}$ is the simplest form or lowest term of $\frac{8}{20}$.

(b) Here, numerator is 21 and denominator is 54.

Let us find the HCF of 21 and 54 by long division method:

$$\begin{array}{r} 21\overline{)\ 54\ (2} \\ -\ 42 \\ \hline 12\overline{)\ 21\ (1} \\ -\ 12 \\ \hline 9\overline{)\ 12\ (1} \\ -\ 9 \\ \hline 3\overline{)\ 9\ (3} \\ -\ 9 \\ \hline 0 \end{array}$$

$\therefore$ HCF of 21 and 54 is 3.

Thus, $\frac{21}{54} = \frac{21 \div 3}{54 \div 3} = \frac{7}{18}$

Hence, $\frac{7}{18}$ is the simplest form or lowest term of $\frac{21}{54}$.

Testing Time 4.4

1. Which of the following fractions are in its lowest term?

(a) $\frac{8}{9}$ (b) $\frac{12}{14}$ (c) $\frac{3}{7}$ (d) $\frac{25}{40}$

2. Reduce each of the following fractions in its lowest term :

(a) $\frac{9}{33}$ (b) $\frac{12}{28}$ (c) $\frac{32}{40}$ (d) $\frac{56}{70}$

(e) $\frac{108}{220}$ (f) $\frac{116}{312}$ (g) $\frac{75}{80}$ (h) $\frac{91}{260}$

(i) $\frac{75}{120}$ (j) $\frac{162}{54}$ (k) $\frac{140}{245}$ (l) $\frac{156}{346}$

TO FIND THE FRACTIONAL PART OF A NUMBER/ QUANTITY

Study the following examples:

Example 6 : Find :

(a) $\frac{2}{3}$ of 18 (b) $\frac{4}{5}$ of 35

Solution : (a) Since, $18 \div 3 = 6$ and $6 \times 2 = 12$

Therefore, $\frac{2}{3}$ of $18 = 12$.

(b) Since, $35 \div 5 = 7$ and $7 \times 4 = 28$

Therefore, $\frac{4}{5}$ of $35 = 28$.

Example 7 : A class has 42 students. $\frac{5}{7}$ of the class went on a picnic. How many students went on the picnic?

Solution : $\frac{5}{7}$ indicates that 42 students is to be divided into 7 equal groups and then 5 groups are taken.

So, first we divide 42 by 7.

$42 \div 7 = 6$

$\therefore$ one group has 6 students.

Now, to find the number of students of 5 groups, we multiply 5 by 6.

$5 \times 6 = 30$

Therefore, 30 students went on the picnic.

Example 8 : Shekhar bought 24 oranges and he ate $\frac{7}{8}$ of them. How many oranges are left with Shekhar now?

Solution : To find the number of oranges left, first we divide 24 by 8, i.e., $24 \div 8 = 3$ and now multiply 3 by 7, i.e., $3 \times 7 = 21$.
So, Shekhar ate 21 oranges.
$\therefore$ Oranges left = $24 - 21 = 3$.
Thus, Shekhar has 3 oranges now.

Testing Time 4.5

1. Determine the value of the following:

(a) $\frac{1}{2}$ of 8 (b) $\frac{2}{3}$ of 24 (c) $\frac{5}{6}$ of 48

(d) $\frac{4}{9}$ of 9 (e) $\frac{11}{8}$ of 16 (f) $\frac{16}{3}$ of 27

(g) $\frac{5}{18}$ of 54 (h) $\frac{2}{25}$ of 50 (i) $\frac{5}{20}$ of 1000

2. Find :

(a) $\frac{5}{5}$ of ₹35 (b) $\frac{9}{5}$ of 50 paise (c) $\frac{7}{12}$ of 420 l

(d) $\frac{3}{8}$ of 40 m (e) $\frac{3}{7}$ of 63 km (f) $\frac{6}{13}$ of 26 kg

(g) $\frac{2}{5}$ of 3 kg (in gram) (h) $\frac{3}{4}$ of a day (in hours) (i) $\frac{9}{4}$ of an hour (in minutes)

3. Fill in the blanks with fractions in lowest terms:

(a) 4 hours is ..... of a day.

(b) 200 g is ..... of 1 kg.

(c) 5 years is ..... of a decade.

(d) 50 paise is ..... of one rupee.

(e) 40 cm is ..... of a metre.

(f) 9 months is ..... of one year.

(g) 15 seconds is ..... of a minute.

4. A class has 56 students. $\frac{4}{7}$ of the class went on a picnic. How many students went on the picnic?

5. Rinku bought 30 balloons. She distributed $\frac{4}{5}$ of the balloons among her friends. How many balloons did she distribute?

6. A shopkeeper bought 550 candles. He found that $\frac{1}{11}$ of the candles were defective and he did not sell them. How many candles did he sell ?

COMPARISON AND ORDER OF FRACTIONS

In previous class, we have learnt comparison of like fractions. We know that of two given like fractions, the fraction with greater numerator is greater. For example,

$\frac{3}{8} < \frac{5}{8}$ or, $\frac{5}{8} > \frac{3}{8}$

$\frac{7}{15} < \frac{8}{15}$ or, $\frac{8}{15} > \frac{7}{15}$

We have also learnt that three or more like fractions can be arranged in ascending or descending order by arranging their numerators in ascending or descending order.

For example, the fractions

$\frac{1}{9}, \frac{5}{9}, \frac{4}{9}, \frac{8}{9}, \frac{7}{9}$ can be arranged in the ascending order as $\frac{1}{9} < \frac{4}{9} < \frac{5}{9} < \frac{7}{9} < \frac{8}{9}$.

Similarly, the fractions

$\frac{3}{14}, \frac{9}{14}, \frac{5}{14}, \frac{11}{14}, \frac{8}{14}$ can be arranged in the descending orders as

$$\frac{11}{14} > \frac{9}{14} > \frac{8}{14} > \frac{5}{14} > \frac{3}{14}.$$

COMPARISON OF FRACTIONS HAVING SAME NUMERATORS AND DIFFERENT DENOMINATORS

If two or more fractions have the same numerator but different denominators, we can compare them as explained below:

Consider two fractions $\frac{4}{6}$ and $\frac{4}{9}$.

Note, that they have the same numerator.

Now, $\frac{4}{6} = \frac{4 \times 3}{6 \times 3} = \frac{12}{18}$

and $\frac{4}{9} = \frac{4 \times 2}{9 \times 2} = \frac{8}{18}$

Since, $12 > 8, \frac{12}{18} > \frac{8}{18}$, i.e., $\frac{4}{6} > \frac{4}{9}$.

What do you conclude?

We conclude that out of two given fractions with same numerator, the fraction with greater denominator is smaller.

Example 9 : Arrange the following fractions in ascending and descending order :

$$\frac{5}{8}, \frac{5}{13}, \frac{5}{6}, \frac{5}{14}, \frac{5}{9}$$

Solution : We arrange the fractions in ascending order or descending by looking at their denominators.

Fractions in ascending order: $\frac{5}{14} < \frac{5}{13} < \frac{5}{9} < \frac{5}{8} < \frac{5}{6}$

Fractions in ascending order: $\frac{5}{6} > \frac{5}{8} > \frac{5}{9} > \frac{5}{13} > \frac{5}{14}$.

COMPARISON OF FRACTIONS WITH UNLIKE NUMERATORS AND UNLIKE DENOMINATORS

When like and unlike fractions are given together, we can also arrange them in ascending or descending order by converting them into like fractions.

Example 10 : Compare $\frac{3}{7}$ and $\frac{5}{6}$.

Solution : Since the given fractions are unlike, we convert them into like fractions

as $\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$; $\frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}$

The fractions $\frac{9}{12}$ and $\frac{10}{12}$ are like fractions.

Clearly, $\frac{9}{12} < \frac{10}{12}$

i.e., $\frac{10}{12} > \frac{9}{12}$.

Example 11 : Arrange the following fractions in ascending order :

$$\frac{2}{3}, \frac{1}{4}, \frac{7}{8}, \frac{5}{6}$$

Solution : The given fractions $\frac{2}{3}, \frac{1}{4}, \frac{7}{8}$ and $\frac{5}{6}$ are unlike fractions.

Therefore, first we must convert them into like fractions. For this we find the LCM of denominators.

LCM of 3, 4, 8 and 6 is 24.

Now, we make the denominators of given fractions 24.

$\frac{2}{3} = \frac{2 \times 8}{3 \times 8} = \frac{16}{24}$; $\frac{1}{4} = \frac{1 \times 6}{4 \times 6} = \frac{6}{24}$

$\frac{7}{8} = \frac{7 \times 3}{8 \times 3} = \frac{21}{24}$; $\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}$

Now, comparing the numerators of the above formed like fractions, we have $6 < 16 < 20 < 21$.

Hence, $\frac{6}{24} < \frac{16}{24} < \frac{20}{24} < \frac{21}{24}$.

Thus, $\frac{1}{4} < \frac{2}{3} < \frac{5}{6} < \frac{7}{8}$ is the required ascending order.

Testing Time 4.6

1. Compare each pair given below using > or < or = :

(a) $\frac{2}{3} \square \frac{1}{2}$ (b) $\frac{1}{4} \square \frac{1}{5}$ (c) $\frac{2}{9} \square \frac{2}{7}$ (d) $\frac{3}{4} \square \frac{4}{9}$

(e) $\frac{5}{9} \square \frac{8}{9}$ (f) $\frac{7}{8} \square \frac{5}{9}$ (g) $\frac{6}{7} \square \frac{6}{11}$ (h) $\frac{1}{5} \square \frac{3}{14}$

(i) $\frac{6}{23} \square \frac{1}{4}$ (j) $\frac{15}{46} \square \frac{18}{49}$ (k) $\frac{11}{18} \square \frac{13}{18}$ (l) $\frac{17}{19} \square \frac{18}{15}$

2. Arrange the following fractions in ascending order :

(a) $\frac{3}{8}, \frac{5}{9}, \frac{2}{4}, \frac{1}{3}, \frac{7}{8}$ (b) $\frac{4}{15}, \frac{3}{15}, \frac{1}{2}, \frac{9}{10}$

(c) $\frac{5}{4}, \frac{5}{8}, \frac{5}{12}, \frac{5}{16}$ (d) $\frac{7}{8}, \frac{7}{15}, \frac{7}{16}, \frac{7}{12}$

(e) $\frac{5}{9}, \frac{3}{10}, \frac{1}{3}, \frac{4}{15}$ (f) $\frac{3}{8}, \frac{1}{16}, \frac{5}{18}, \frac{7}{4}$

3. Arrange the following fractions in descending order :

(a) $\frac{7}{9}, \frac{3}{9}, \frac{1}{9}, \frac{5}{9}, \frac{2}{9}$ (b) $\frac{13}{30}, \frac{9}{30}, \frac{11}{30}, \frac{17}{30}, \frac{15}{30}$

(c) $\frac{8}{15}, \frac{2}{15}, \frac{14}{15}, \frac{7}{15}, \frac{1}{15}$ (d) $\frac{12}{17}, \frac{11}{17}, \frac{9}{17}, \frac{16}{17}, \frac{8}{17}$

(e) $\frac{3}{8}, \frac{3}{5}, \frac{3}{6}, \frac{3}{4}$ (f) $\frac{1}{4}, \frac{1}{2}, \frac{1}{8}, \frac{3}{7}$

ADDITION OF FRACTIONAL NUMBERS

In class IV, we have learnt the addition of like fractions. In this class, we shall learn the addition of unlike fractions.

REVISION – ADDITION OF LIKE FRACTIONS

Rule. To add two or more like fractions, we add the numerators of the given fractions. The number thus obtained becomes the numerator of the required fraction and the denominator of this fraction is the common denominator of the given fractions.

i.e., Sum of like fractions $= \frac{\text{Sum of numerators}}{\text{Common denominators}}$.

Example 12 : Add :

(a) $\frac{1}{5} + \frac{3}{5}$ (b) $\frac{2}{15} + \frac{4}{15} + \frac{7}{15}$

Solution : (a) $\frac{1}{5} + \frac{3}{5} = \frac{1+3}{5} = \frac{4}{5}$

(b) $\frac{2}{15} + \frac{4}{15} + \frac{7}{15} = \frac{2+4+7}{15} = \frac{13}{15}$.

ADDITION OF UNLIKE FRACTIONS

Rule. To add unlike fractions, we first change them into like fractions. For this, we find corresponding equivalent fractions having the same denominator and then we add them as usual.

Example 13 : Add : $\frac{2}{3} + \frac{4}{5}$.

Solution : $\frac{2}{3}$ and $\frac{4}{5}$ are unlike fractions.

Let us find their equivalent like fractions.

The L.C.M. of the denominators of 3 and 5 is 15.

Now, we convert the fractions into equivalent fractions with denominator 15.

$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$ (since $15 \div 3 = 5$)

and $\frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$ (since $15 \div 5 = 3$)

Thus, $\frac{2}{3} + \frac{4}{5} = \frac{10}{15} + \frac{12}{15}$

$= \frac{10 + 12}{15}$

$= \frac{22}{15}$.

In view of the above, when we add unlike fractions, we take the following steps:

Step 1. Find the LCM of denominators of unlike fractions.

Step 2. Convert unlike fractions into equivalent like fractions with their LCM as common denominator.

Step 3. Add like fractions so obtained.

Example 14 : Solve:

(a) $\frac{1}{3} + \frac{1}{4} + \frac{1}{6}$

(b) $\frac{5}{6} + \frac{4}{9} + \frac{3}{4}$

Solution : (a) $\frac{1}{3} + \frac{1}{4} + \frac{1}{6} = \frac{1 \times 4}{3 \times 4} + \frac{1 \times 3}{4 \times 3} + \frac{1 \times 2}{6 \times 2}$

$= \frac{4}{12} + \frac{3}{12} + \frac{2}{12}$ (LCM of 3, 4 and 6 is 12.)

$= \frac{4 + 3 + 2}{12}$

$= \frac{9}{12}$

$= \frac{3}{4}$

We write the answer as a mixed number only if asked specifically.

(b) $\frac{5}{6} + \frac{4}{9} + \frac{3}{4} = \frac{5 \times 6}{6 \times 6} + \frac{4 \times 4}{9 \times 4} + \frac{3 \times 9}{4 \times 9}$ (LCM of 6, 9 and 4 is 36.)

$= \frac{30}{36} + \frac{16}{36} + \frac{27}{36}$

$= \frac{30 + 16 + 27}{36} = \frac{73}{36}$.

Testing Time 4.7

1. Find the equivalent fractions in each of the following with lowest common denominator:

(a) $\frac{3}{4}$ and $\frac{7}{8}$ (b) $\frac{5}{8}$ and $\frac{2}{9}$ (c) $\frac{4}{9}$ and $\frac{7}{12}$ (d) $\frac{8}{35}$ and $\frac{6}{14}$

2. Add:

(a) $\frac{2}{3}$ and $\frac{4}{7}$ (b) $\frac{4}{5}$ and $\frac{3}{7}$ (c) $\frac{3}{4}$ and $\frac{1}{6}$ (d) $\frac{3}{5}$ and $\frac{2}{7}$

(e) $\frac{2}{10}$ and $\frac{3}{5}$ (f) $\frac{3}{10}$ and $\frac{2}{25}$ (g) $\frac{8}{10}$ and $\frac{2}{3}$ (h) $\frac{3}{11}$ and $\frac{1}{5}$

3. Find the sum:

(a) $\frac{2}{3}+\frac{4}{5}+\frac{1}{2}$ (b) $\frac{7}{10}+\frac{8}{15}+\frac{3}{5}$ (c) $\frac{5}{8}+\frac{9}{16}+\frac{13}{24}$ (d) $\frac{3}{4}+\frac{5}{12}+\frac{7}{8}$

(e) $\frac{13}{50}+\frac{9}{10}+\frac{8}{20}$ (f) $\frac{3}{4}+\frac{2}{3}+\frac{23}{24}$ (g) $\frac{5}{6}+\frac{1}{4}+\frac{2}{5}$ (h) $\frac{5}{7}+6+\frac{12}{21}$

(i) $\frac{2}{3}+\frac{6}{7}+\frac{11}{28}$ (j) $7+\frac{7}{6}+\frac{5}{14}$

SUBTRACTION OF FRACTIONAL NUMBERS

In class IV, we have learnt to find the difference between two like fractions. Now, we shall learn to find difference between two unlike fractions.

REVISION – SUBTRACTION OF LIKE FRACTIONS

Rule : To subtract a like fraction from another like fraction, we subtract the smaller numerator from the greater numerator. The number thus obtained is the numerator of the required fraction and the denominator of this fraction is the common denominator of the given fractions.

i.e., Difference of like fractions $= \frac{\text{Difference between the numerators}}{\text{Common denominator}}$

Example 15: Subtract $\frac{4}{9}$ from $\frac{5}{9}$.

Solution : $\frac{5}{9}-\frac{4}{9}=\frac{5-4}{9}=\frac{1}{9}$

SUBTRACTION OF UNLIKE FRACTIONS

Rule. To subtract a fraction from another fraction of unlike denominators, we change both the fractions to equivalent fractions of common denominator and then subtract as usual.

Example 16: Subtract $\frac{5}{8}$ from $\frac{13}{16}$.

Solution : LCM of 8 and 16 is 16.

So, we change the given fractions to equivalent fractions with 16 as denominator.

Now, $\frac{5}{8} = \frac{5 \times 2}{8 \times 2}$

$= \frac{10}{16}$

$\therefore \frac{13}{16} - \frac{5}{8} = \frac{13}{16} - \frac{10}{16}$

$= \frac{13 - 10}{16}$

$= \frac{3}{16}$.

Example 17: Find the difference between $\frac{14}{15}$ and $\frac{19}{25}$

Solution : LCM of 15 and 25 is 75.

So, we change the fractions to equivalent fractions with 75 as denominator.

Now, $\frac{14}{15} = \frac{14 \times 5}{15 \times 5}$

$= \frac{70}{75}$

and $\frac{19}{25} = \frac{19 \times 3}{25 \times 3}$

$= \frac{57}{75}$

Since $70 > 57$, so $\frac{70}{75} > \frac{57}{75}$.

$\therefore \frac{70}{75} - \frac{57}{75} = \frac{70 - 57}{75} = \frac{13}{75}$.

Testing Time 4.8

1. Solve :

(a) $\frac{5}{7} - \frac{6}{11}$ (b) $\frac{6}{13} - \frac{1}{4}$ (c) $\frac{3}{7} - \frac{5}{12}$

(d) $\frac{21}{28} - \frac{1}{4}$ (e) $1 - \frac{5}{12}$ (f) $4 - \frac{6}{10}$

2. Subtract :

(a) $\frac{2}{5}$ from $\frac{4}{9}$ (b) $\frac{7}{20}$ from $\frac{9}{15}$ (c) $\frac{5}{12}$ from $\frac{7}{8}$

(d) $\frac{3}{8}$ from $\frac{5}{9}$ (e) $\frac{1}{42}$ from $\frac{1}{24}$ (f) $\frac{4}{13}$ from 2

3. Find the difference between:

(a) $\frac{5}{4}$ and $\frac{4}{5}$ (b) $\frac{5}{6}$ and $\frac{6}{7}$ (c) $\frac{2}{3}$ and $\frac{4}{7}$

(d) $\frac{2}{7}$ and $\frac{4}{21}$ (e) $\frac{1}{4}$ and $\frac{3}{13}$ (f) $\frac{5}{18}$ and $\frac{4}{15}$

(g) $\frac{4}{9}$ and $\frac{7}{25}$ (h) 4 and $\frac{23}{12}$ (i) $\frac{15}{22}$ and $\frac{7}{18}$

MULTIPLICATION OF FRACTIONAL NUMBERS

To multiply two or more fractional numbers, we multiply their numerators together and denominators together separately as explained below:

$$\frac{4}{5} \times \frac{2}{7}$$

$$= \frac{4 \times 2}{5 \times 7} = \frac{8}{35};$$

$$\frac{1}{6} \times \frac{3}{4} = \frac{1 \times 3}{6 \times 4}$$

$$= \frac{3}{24} = \frac{1}{8}$$

Thus, to multiply two or more fractional numbers, we take the following steps:

Step 1. Multiply the numerators of the given fractions.

Step 2. Multiply the denominators of the given fractions.

Step 3. Form the fraction whose numerator is the number obtained in Step 1 and the denominator is the number obtained in Step 2.

This fraction is the required product of the given fractional numbers.

Example 18 : Multiply $\frac{3}{7}$ by $\frac{5}{9}$.

Solution : $\frac{3}{7} \times \frac{5}{9} = \frac{3 \times 5}{7 \times 9}$

$= \frac{15}{63}$

$= \frac{5}{21}$.

Sometimes, for indicating multiplication of two fractional numbers, we use the word 'of ' such as

$\frac{1}{2}$ of $\frac{1}{3}$ for $\frac{1}{3} \times \frac{1}{2}$.

i.e., $\frac{1}{2}$ of $\frac{1}{3}$ means $\frac{1}{3} \times \frac{1}{2} = \frac{1 \times 1}{3 \times 2} = \frac{1}{6}$.

Example 19 : Find $\frac{3}{8}$ of 40.

Solution : $\frac{3}{8}$ of $40 = 40 \times \frac{3}{8}$

$= \frac{40}{1} \times \frac{3}{8}$

$= \frac{40 \times 3}{1 \times 8}$

$= \frac{120}{8}$

$= 15$.

DIVISION OF FRACTIONAL NUMBERS

Before we discuss the division of fractional numbers, let us learn the reciprocal or multiplicative inverse of a number.