Chapter 6
Multiples and Factors

In the previous class, we have learnt about multiples, factors, composite and prime numbers. Here in this class, we shall review these concepts and the properties related to them.

MULTIPLES

The product of two or more numbers is said to be a multiple of each of these numbers.

For example, $5 \times 7 = 35$

The product of 5 and 7 is 35. So, 35 is a multiple of both the numbers 5 and 7.

Example 1 : Write the first four multiples of 9.

Solution : To find the first four multiples of 9, we multiply 9 by the first four counting numbers.

$9 \times 1 = 9$ $\qquad$ $9 \times 2 = 18$

$9 \times 3 = 27$ $\qquad$ $9 \times 4 = 36$

Thus, the first four multiples of 9 are 9, 18, 27, 36.

PROPERTIES OF MULTIPLES

1. Every number is a multiple of itself.
   For example, $4 \times 1 = 4$, $19 \times 1 = 19$, $258 \times 1 = 258$
   Therefore, 4 is a multiple of 4, 19 is a multiple of 19, 258 is a multiple of 258 and so on.
2. Every number is a multiple of 1.
   For example, $1 \times 6 = 6$, $1 \times 73 = 73$ and so on.
3. Every multiple of a number is exactly divisible by the number itself.

FACTORS

When two or more numbers are multiplied to give a product, each number is called the **factor of the product**.

Let us consider the number 24.

We know that, $4 \times 6 = 24$.

$\therefore$ 4 and 6 are factors of 24.

We can also see that $24 \div 4 = 6$ and $24 \div 6 = 4$.

Hence, we can also say that ' a factor of a number is a number which exactly divides it.'

Let us now consider the number 72.

Each of the numbers 1, 2, 3, 4, 6, 9, 12, 18, 36 and 72 are factors of 72 because they divide 72 exactly.

PROPERTIES OF FACTORS

1. **1 is a factor of every whole number.**

   We know that $1 \times 4 = 4$, $1 \times 6 = 6$, $1 \times 25 = 25$ etc

   So, 1 is a factor of every number.

2. **Every whole number (except 0) is a factor of itself.**

   We know that $1 \times 5 = 5$, $9 \times 1 = 9$, $22 \times 1 = 22$ etc

   So, every non zero-number is a factor of itself.

3. **Every whole number except 1 has at least two factors.**

   We know that, $2 = 1 \times 2$, $10 = 2 \times 5$, $30 = 2 \times 3 \times 5$

   So, every whole number except one has at least two factors.

Example 2 : Find all the factors of 48.

Solution : The multiplication facts of 48 are

$48 = 1 \times 48 = 2 \times 24 = 3 \times 16 = 4 \times 12 = 6 \times 8$

Therefore, the factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24 and 48.

Example 3 : Check whether 17 is a factor of 25741 or not.

Solution : To check whether 17 is a factor of 25741, we divide 25741 by 17.

```
        1514
   17) 25743
      -17
       ---
        87
      - 85
       ----
         24
       - 17
        ----
          73
        - 68
         ----
           5
```

Since, the remainder is not zero, 17 is not a factor of 25743.

Example 4 : Check whether 14 is a factor of 86268 or not.

Solution :

```
      6162
14 ) 86268
     84
     --
      22
      14
      --
       86
       84
       --
        28
        28
        --
         0
```

Since, the remainder is zero, 14 is a factor of 86268.

Remember : A multiple of a number is always greater than or equal to the number.

EVEN AND ODD NUMBERS

A number which is a multiple of 2 is called an **even number**.

For example, 2, 4, 6, 8, ... are even numbers.

A number which is not a multiple of 2 is called an **odd number**.

For example 1, 3, 5, 7, ... are odd numbers.

COMPOSITE AND PRIME NUMBERS

Composite Numbers : All the numbers which have more than two factors are called composite numbers.

For example, 4, 6, 8, 9, 10, 12, 14, 15, ... are composite numbers.

Prime Numbers : The numbers which have only two factors, i.e.,1 and the number itself are called prime numbers.

For example, 2, 3, 5, 7, 11, ... are prime numbers.

Prime numbers less than 100 are :

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

So, there are 25 prime numbers and 74 composite numbers less than 100.

Remember : Since the number 1 has only one factor, it is neither a prime nor a composite number.

2 is the smallest prime number.

All even numbers are composite numbers.

2 is the only even prime number, all other prime numbers are odd.

PRIME FACTORIZATION

When we express a number as a product of its factors, we say that we have factorized the number. For example, 24 can be factorized as follows :

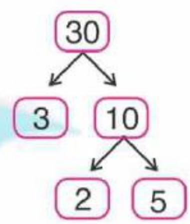
$24 = 2 \times 12$, $\quad 24 = 3 \times 8$, $\quad 24 = 2 \times 2 \times 2 \times 3$

These factorizations are of different types. In the first and second factorizations, one factor is prime and the another factor is a composite number. However, in third factorization, all the factors are prime numbers.

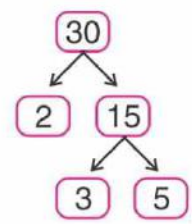
The factorization of the type where all the factors are prime numbers is called a **prime factorization**.

From this discussion of factorization, we can say that every composite number has one and only one prime factorization.

Consider the number 30. We find its factor with this method as under :



$30 = 3 \times 2 \times 5$



$30 = 2 \times 3 \times 5$

So, the number 30 can be expressed as a product of factors which are all prime numbers. From the two ways of listing the factors of 30, we see that prime factors of a number are always the same except for the order of writing them.

The following examples will illustrate the same.

Example 5 : Write the prime factorization of 36.

Solution : Let us factorize the number by division method.

2	36
2	18
3	9
3	3
	1

Thus, the prime factorization of 36 is $2 \times 2 \times 3 \times 3$.

Testing Time 6.1

1. **Write first four multiples of the following numerals:**
   (a) 6 (b) 19 (c) 24 (d) 32 (e) 41
2. **Write all the factors of the following numerals:**
   (a) 48 (b) 104 (c) 160 (d) 216 (e) 385
3. Write all the odd numbers between 351 and 369.
4. Write all the even numbers between 215 and 231.
5. **Write the prime factorization of the following numerals:**
   (a) 42 (b) 54 (c) 96 (d) 258 (e) 693
6. **In each of the following cases, state whether the first number is a factor of the second number:**
   (a) 8, 256 (b) 13, 826 (c) 15, 690 (d) 21, 468

7. Find a 3-digit number which is the sum of the smallest prime number and greatest 2-digit composite number.
8. Look at the following pairs of numbers and determine whether they are co-prime or twin prime:
   (a) 8, 9 (b) 11, 13 (c) 15, 16 (d) 64, 65 (e) 71, 73

HIGHEST COMMON FACTOR (HCF)

In class IV, we have studied that the greatest of the common factors of two or more than two whole numbers is called their highest common factor (HCF).

Let us consider three numbers 12, 16 and 28.

Factors of 12 are 1, 2, 3, 4, 6, 12.

Factors of 16 are 1, 2, 4, 8, 16.

Factors of 28 are 1, 2, 4, 7, 14, 28.

Common factors of 12, 16, and 28 are 1, 2, 4.

Highest common factor of 12, 16 and 28 is 4.

$\therefore$ HCF = 4

Also, $28 \div 4 = 7$, $16 \div 4 = 4$, $12 \div 4 = 3$.

We see that the HCF of 12, 16, and 28 divides them exactly.

Hence, we can say that 'the HCF of two or more numbers is the largest number that divides them exactly.

We shall use this definition in solving word problems on HCF later.

Note : The highest common factor is also called the greatest common divisor or G. C. D.

METHODS OF FINDING THE HCF

We can find the HCF of two or more numbers by the following two methods :

1. Prime Factorization Method 2. Long Division Method

1. Prime Factorization Method: This method is used to find the HCF of smaller numbers whose prime factorization can be easily worked out.

In this method, we take the following steps:

Step 1. Write the prime factorization of each of the given numbers.

Step 2. Identify the prime factors which are common to all numbers.

Step 3. Find the product of the common factors by taking each common prime factor the least number of times it appears in the prime factorization of all the numbers.

The product so obtained is the required HCF.

Example 6 : Find the HCF of 24 and 42 by prime factorization method.

Solution : Resolving each of the given numbers into prime factors, we get

2	24
2	12
2	6
3	3
	1

2	42
3	21
7	7
	1

Prime factors of 24 and 42 are

$$24 = 2 \times 2 \times 2 \times 3$$
$$42 = 2 \times 3 \times 7$$

Common factors are 2 and 3. The smallest power of 2 is 1 and 3 has also 1.

$\therefore$ HCF of 24 and 42 is $2 \times 3 = 6$.

2. Long Division Method : This method is used to find the HCF of large numbers whose prime factorization is not easy to workout. In this method, we take the steps given on next page.

Step 1. Take the greater number as dividend and the smaller number as divisor.

Step 2. Find the quotient and remainder. If the remainder is zero then the divisor is the required HCF, otherwise the remainder is taken as the new divisor and the divisor as the new dividend.

Step 3. Repeat this division till the remainder is zero. The last divisor is the required HCF of the two given numbers.

If we have to find the HCF of three numbers, we first find the HCF of any two of them by the same process as explained above and then take the HCF of these two numbers and the third number and repeat the above process to find the required HCF.

Example 7 : Find the HCF of 21 and 48 by the long division method.

Solution :

```
21) 48 (2
  − 42
  ------
     6) 21 (3
      − 18
      ------
         3) 6 (2
          − 6
          ---
            0
```

Hence, the HCF of 21 and 48 is 3.

Example 8 : Find the HCF of 128 and 304 by the long division method.

Solution :

```
128) 304 (2
   - 256
   -----
     48) 128 (2
       - 96
       -----
         32) 48 (1
           - 32
           -----
             16) 32 (2
                -32
                ----
                  0
```

Hence, the HCF of 128 and 304 is 16.

Testing Time 6.2

1. Find the HCF of the following set of numbers by the prime factorization method :

(a) 15 and 27 (b) 48 and 84 (c) 55 and 70

(d) 72 and 96 (e) 124 and 142 (f) 136 and 154

(g) 24, 72 and 92 (h) 32, 48 and 144 (i) 60, 140 and 240

2. Find the HCF of the following numbers using the long division method :

(a) 12, 44 (b) 17, 61 (c) 26, 84 (d) 40, 90

(e) 79, 91 (f) 82, 96 (g) 90, 115 (h) 36, 162

(i) 39, 78, 182 (j) 52, 88, 124 (k) 57, 209, 399

LOWEST COMMON MULTIPLE (LCM)

In class IV, we have studied that the smallest of the common multiples of two or more whole numbers is called their **lowest common multiple** (LCM).

Let us consider three numbers 6, 8 and 12.

Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, ...

Multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, ...

Multiples of 12 are 12, 24, 36, 48, 60, 72, 84, 96, ...

Common multiples of 6, 8 and 12 are 24, 48, ...

Lowest common multiple of 6, 8 and 12 is 24.

$\therefore$ LCM of 6, 8 and 12 = 24.

Also, $24 \div 6 = 4$, $24 \div 8 = 3$, $24 \div 12 = 2$.

We see that the LCM of 6, 8 and 12 is exactly divisible by them.

Hence, we can say that 'the LCM of two or more numbers is the smallest number which is exactly divisible by them. We shall use this definition in solving word problems on LCM later.

METHOD OF FINDING THE LCM

We can find the LCM of two or more numbers by the following two methods:

1. Prime Factorization Method
2. Division Method

1. Prime Factorization Method: This method is used for finding the LCM of small numbers whose prime factorization can be easily worked out.

In this method we take the following steps:

Step 1. Write the prime factorization of each of the given numbers.

Step 2. Take each prime factor the maximum number of times it occurs in the obtained factorization.

Step 3. Find the product of the prime factors obtained in step 2. This product is the required LCM.

Example 9 : Find the L. C. M. of 18 and 32 by the prime factorization method.

Solution :

2	18
3	9
3	3
	1

2	32
2	16
2	8
2	4
2	2
	1

Prime factorizations of 18 and 32 are

$18 = 2 \times 3 \times 3$

$32 = 2 \times 2 \times 2 \times 2 \times 2$

2 occurs maximum 5 times and 3 occurs maximum 2 times.

$\therefore$ LCM of 18 and 32 $= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 288$.

Example 10 : Find the LCM of 24, 42 and 68 by the prime factorization.

Solution :

2	24
2	12
2	6
3	3
	1

2	42
3	21
7	7
	1

2	68
2	34
17	17
	1

Prime factorizations of 24, 42 and 68 are

$24 = 2 \times 2 \times 2 \times 3$ $42 = 2 \times 3 \times 7$ $68 = 2 \times 2 \times 17$

2 occurs maximum 3 times, 3, 7 and 17 occur only one time.

$\therefore$ LCM of 24, 42 and 68 $= 2 \times 2 \times 2 \times 3 \times 7 \times 17 = 2856$.

2. Division Method : This method is used to find the LCM of large numbers whose prime factorization is not easy to workout.

In this method, we take the following steps:

Step 1. Write the given numbers in a row and find out a number which divides exactly at least two of the given numbers.

Step 2. Divide the numbers which are exactly divisible by the chosen number and write their quotient below them. Carry forward the number (or numbers) which is not divisible.

Step 3. Repeat the step-3 you get the numbers which are prime to one another.

Step 4. The product of all the divisors and the numbers obtained in the last row is the required LCM.

Example 11 : Find the LCM of 64 and 120 by the division method.

Solution :

2	64,	120
2	32,	60
2	16,	30
	8,	15

Here, the product of divisors $= 2 \times 2 \times 2 = 8$ and

The product of the undivided numbers $= 8 \times 15 = 120$.

$\therefore$ The required LCM $= 8 \times 120 = 960$.

Example 12 : Find the LCM of 56, 84 and 128.

Solution :

2	56,	84,	128
2	28,	42,	64
2	14,	21,	32
7	7,	21,	16
	1,	3,	16

Here, the product of divisors $= 2 \times 2 \times 2 \times 7 = 56$ and

The product of the numbers in the last row $= 1 \times 3 \times 16 = 48$.

$\therefore$ The required LCM $= 56 \times 48 = 2688$.

Testing Time 6.3

1. Find the LCM of each of the following set of numbers using prime factorization method:

(a) 14, 21 (b) 32, 96 (c) 28, 60

(d) 58, 62 (e) 56, 32 (f) 55, 85

(g) 9, 15, 36 (h) 18, 52, 75 (i) 16, 76, 264

2. Find the LCM of the following using the division method:

(a) 48, 84 (b) 9, 36, 48 (c) 16, 72, 84

(d) 25, 40, 75 (e) 63, 98, 112 (f) 108, 75, 230

(g) 90, 81, 279 (h) 256, 308, 528 (i) 125, 650, 500

RELATIONSHIP BETWEEN THE LCM AND HCF OF TWO NUMBERS

Let us consider the numbers 12 and 30.

2	12
2	6
3	3
	1

2	30
3	15
5	5
	1

Prime factorization of $12 = 2 \times 2 \times 3$

Prime factorization of $30 = 2 \times 3 \times 5$

HCF of 12 and $30 = 2 \times 3 = 6$

LCM of 12 and $30 = 2 \times 2 \times 3 \times 5 = 60$

Product of HCF and LCM $= 6 \times 60 = 360$

Product of the two numbers $= 12 \times 30 = 360$

We see that the two products are equal.

Thus, we get a very important property of HCF and LCM of two numbers.

The product of two numbers = The product of thier HCF and LCM

or, $$\text{HCF} = \frac{\text{Product of the two numbers}}{\text{LCM}}$$

or, $$\text{LCM} = \frac{\text{Product of the two numbers}}{\text{HCF}}$$

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Example 13 : The product of two numbers is 468 and their LCM is 78, find their HCF.

Solution : We know that :

$$\text{HCF} = \frac{\text{Product of the two numbers}}{\text{LCM}}$$

$$\therefore \text{ Required HCF} = \frac{468}{78} = 6.$$

APPLICATIONS OF HCF AND LCM

We have learnt about LCM and HCF of two or more numbers. The properties of LCM and HCF help us in solving problems faced by us in our daily life. Let us take some examples.

Example 14 : Find the greatest number which can divide 54, 116 and 260 exactly.

Solution : Required number is HCF of 54, 116 and 260.

```
54 ) 116 ( 2
   – 108
   -------
      8 ) 54 ( 6
        – 48
        -------
          6 ) 8 ( 1
            – 6
            -------
              2 ) 6 ( 3
                – 6
                -----
                  0
```

```
2 ) 260 ( 130
  – 2
  -----
     6
   – 6
   -----
     0
```

HCF of 54, 116, 260 = 2.

Thus, the required number is 2.

Example 15 : Find the smallest number which is exaclty divisible by 18, 45 and 54.

Solution : Required number is LCM of 18, 45 and 54.

2	18, 45, 54
3	9, 45, 27
3	3, 15, 9
	1, 5, 3

LCM of 18, 45 and 54 = $2 \times 3 \times 3 \times 5 \times 3 = 270$.

Thus, the required number is 270.

Example 16 : Two ropes 15 m and 35 m long are to be cut into small pieces of equal lengths. Find the maximum length of each piece that can be measured exactly.

Solution : Length of required piece = HCF of 15 m and 35 m.

```
15) 35 (2
  – 30
  ------
      5) 15 (3
       – 15
       -----
          0
```

Thus, the maximum length of each piece = 5 m.

Testing Time 6.4

1. **In each of the following pair of numbers, verify that the product of numbers is equal to the product of LCM and HCF :**

   (a) 32 and 46 (b) 58 and 76 (c) 115 and 250

2. The product of two numbers is 504 and their HCF is 8. Find the LCM of the two numbers.
3. Find the HCF of two numbers so that their product is 2754 and their LCM is 162.
4. The LCM and HCF of two numbers are 256 and 6 respectively. If one number is 96, find the other.
5. Find the greatest number which can divide 30, 45 and 90 exactly.
6. Find the greatest number which can divide 62, 78 and 96 exactly.
7. A shopkeeper sold copies of the same variety for ₹ 56, ₹ 70 and ₹112 on three consecutive days. Find the maximum price of each copy.
8. Find the smallest number which is exactly divisible by 8, 20 and 24.
9. Find the smallest number which is exactly divisible by 36, 60 and 84.
10. Three bells ring at intervals of 12,15 and 20 minutes. If they all ring at 9 a.m. together, then find time when they will ring together.