Mathematics — Ganita Manjari

Chapter 11 Test 7

The World of Algorithms

Class IX | 2026–27

Time Allowed: 45 Minutes | Maximum Marks: 25

General Instructions

1. All 13 questions are compulsory. Choose one option in A; show working in B–D.
2. No calculators. Give exact answers unless an approximation is requested.
3. Unless stated otherwise, inputs are positive integers. Give precise algorithm steps.

SECTION A

(5 questions × 1 mark = 5 marks)

1. If $m = qn + r$ with $n > 0$ is a division-with-remainder statement, which condition must hold?

(A) $0 \le r < n$ (B) $r \ge n$

(C) $r < 0$ (D) $r = q$

2. In $71 = 5 \times 13 + 6$, which number replaces 71 after the divisor 13 becomes the first entry of the next GCD pair?

(A) 5 (B) 13

(C) 6 (D) 65

3. The last division in a GCD calculation is $45 = 3 \times 15 + 0$. The GCD is:

(A) 0 (B) 3

(C) 45 (D) 15

4. Why can one remainder calculation replace several subtractions of n from m?

(A) It changes the common divisors

(B) It batches repeated subtractions

(C) It always makes the answer 1

(D) It avoids a stopping condition

5. If the only positive common divisor of two inputs is 1, the inputs are called:

(A) Equal (B) Both prime

(C) Coprime (D) Both even

SECTION B

(4 questions × 2 marks = 8 marks)

6. A student writes $83 = 4 \times 17 + 15$ and then $17 = 0 \times 15 + 17$. Which line is invalid as a division step? Correct it.

7. A GCD calculation begins $391 = 1 \times 299 + 92$ and $299 = 3 \times 92 + 23$. Complete it and state the original GCD.

8. In the remainder algorithm, suppose the first input is smaller: $(m, n) = (7,20)$. Show the first two reductions if no initial swap is made. Is this start still valid?

9. For $101 = 6 \times 16 + 5$, how many subtractions of 16 are compressed into this division step? State the resulting GCD pair and explain why another subtraction is not included.

SECTION C

(3 questions × 3 marks = 9 marks)

10. Let $m = qn + r$ with $n > 0$ and $0 \leq r < n$. Prove that $gcd(m, n) = gcd(n, r)$.

11. A student claims $gcd(m, n) = gcd(m, m \bmod n)$. Test this with $m = 12, n = 5$, and state the correct reduction.

12. Explain why repeated remainder reductions on non-negative integers, not both zero, must terminate when the second input becomes zero.

SECTION D

(1 application question × 3 marks = 3 marks)

13. Use the division algorithm to reduce the fraction $\frac{462}{1078}$ to lowest terms. Explain why no further common factor remains.