Exercise 1.1

Choose the correct answer from the given four options in the following questions:

Question 3:

$n^2 - 1$ *is divisible by* 8, *if* n *is*

(A) an integer
(B) a natural number
(C) an odd integer
(D) an even integer

Answer 3:

(C) an odd integer

Solution:

Let $a = n^2 - 1$

Here n can be even or odd.

Case I

n = Even i.e., n = 2k, where k is an integer.

$\Rightarrow a = (2k)^2 - 1$

$\Rightarrow a = 4k^2 - 1$

At $k = -1$,

$a = 4(-1)^2 - 1 = 4 - 1 = 3$, which is not divisible by 8.

At $k = 0$,

$a = 4(0)^2 - 1 = 0 - 1 = -1$, which is not divisible by 8.

Case II

n = odd i.e., $n = 2k + 1$, where k is an integer

$\Rightarrow a = (2k + 1)^2 - 1$

$\Rightarrow a = 4k^2 + 4k + 1 - 1$

$\Rightarrow a = 4k^2 + 4k$

$\Rightarrow a = 4k(k + 1)$

At $k = -1$,

$a = 4(-1)(-1 + 1) = 0$ which is divisible by 8.

At $k = 0$,

$a = 4(0)(0 + 1) = 0$ which is divisible by 8.

At $k = 1$,

$a = 4(1)(1 + 1) = 8$ which is divisible by 8.

Hence, we can conclude from above two cases, if n is odd, then $n^2 - 1$ is divisible by 8.