Exercise 1.3

Question 1:

Show that the square of any positive integer is either of the form 4q or 4q + 1 for some integer q.

Answer 1:

Let a be an arbitrary positive integer. Then, by Euclid's division algorithm, corresponding to the positive integers a and 4, there exist non – negative integers m and r, such that

$a = 4m + r,\ \ where\ \ 0 \leq r < 4$

$\Longrightarrow a^2 = (4m + r)^2$

$= 16m^2 + r^2 + 8mr$ …. (i)

Where, $0 \leq r < 4$

Case I

When r = 0, then putting r = 0 in Equation (i), we get

$a^2 = 16m^2 = 4(4m^2) = 4q$

Where, $q = 4m^2\ is\ an\ integer$

Case II

When r = 1, then putting r=1 in Equation (i), we get

$a^2 = 16m^2 + 1 + 8m$

$= 4(4m^2 + 2m) + 1 = 4q + 1$

Where, q = $4(4m^2 + 2m)$ is an integer.

Case III

When r = 2, then putting r = 2 in Equation (i), we get

$a^2 = 16m^2 + 4 + 16m$

$= 4(4m^2 + 4m + 1) = 4q$

Where, $q = (4m^2 + 4m + 1)$ *is an integer*

Case IV

When r = 3, then putting r = 3 in Equation (i), we get

$a^2 = 16m^2 + 924m$

$= 16m^2 + 24m + 8 + 1$

$= 4(4m^2 + 6m + 2) + 1$

$= 4q + 1$

Where, $q = (4m^2 + 6m + 2)$ is an integer

Hence, the square of any positive integer is either of the form 4q or 4q + 1 from some integer q.