

Mathematics

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(Chapter – 1) (Real Numbers)(Exemplar Problems)
(Class – X)

Exercise 1.3

Question 1:

Show that the square of any positive integer is either of the form $4q$ or $4q + 1$ for some integer q .

Answer 1:

Let a be an arbitrary positive integer. Then, by Euclid's division algorithm, corresponding to the positive integers a and 4 , there exist non – negative integers m and r , such that

$$a = 4m + r, \text{ where } 0 \leq r < 4$$

$$\Rightarrow a^2 = (4m + r)^2$$

$$= 16m^2 + r^2 + 8mr \quad \dots (i)$$

Where, $0 \leq r < 4$

Case I



When $r = 0$, then putting $r = 0$ in Equation (i), we get

$$a^2 = 16m^2 = 4(4m^2) = 4q$$

Where, $q = 4m^2$ is an integer

Case II

When $r = 1$, then putting $r=1$ in Equation (i), we get

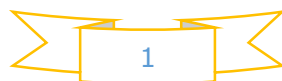
$$a^2 = 16m^2 + 1 + 8m$$

$$= 4(4m^2 + 2m) + 1 = 4q + 1$$

Where, $q = 4(4m^2 + 2m)$ is an integer.

Case III

When $r = 2$, then putting $r = 2$ in Equation (i), we get



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$$\begin{aligned}a^2 &= 16m^2 + 4 + 16m \\ &= 4(4m^2 + 4m + 1) = 4q\end{aligned}$$

Where, $q = (4m^2 + 4m + 1)$ is an integer

Case IV

When $r = 3$, then putting $r = 3$ in Equation (i), we get

$$\begin{aligned}a^2 &= 16m^2 + 924m \\ &= 16m^2 + 24m + 8 + 1 \\ &= 4(4m^2 + 6m + 2) + 1 \\ &= 4q + 1\end{aligned}$$

Where, $q = (4m^2 + 6m + 2)$ is an integer

Hence, the square of any positive integer is either of the form $4q$ or $4q + 1$ from some integer q .

