Exercise 1.3

Question 14:

Prove that $\sqrt{p}+\sqrt{q}$ is irrational, where p and q are primes.

Answer 14:

Let us suppose that $\sqrt{p}+\sqrt{q}$ is rational

Again, let $\sqrt{p}+\sqrt{q}=a$, where a is rational,

Therefore, $\sqrt{q}=a-\sqrt{p}$

On squaring both sides, we get

$q=a^2+p-2a\sqrt{p}$ $[\because (a-b)^2=a^2+b^2-2ab]$

Therefore, $\sqrt{p}=\frac{a^2+p-q}{2a}$,

Which is a contradiction as the right hand side is rational number, while left hand side $\sqrt{p}$ is irrational, since p and q are prime numbers.

Hence, $\sqrt{p}+\sqrt{q}$ is irrational.