

Mathematics

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(Chapter – 1) (Real Numbers)(Exemplar Problems)
(Class – X)

Exercise 1.3

Question 2:

Show that cube of any positive integer is of the form $4m$, $4m + 1$ or $4m + 3$, for some integer m .

Answer 2:

Let a be an arbitrary positive integer. Then, by Euclid's division algorithm, corresponding to the positive integers a and 4 , there exist non-negative integers q and r such that

$$a = 4q + r, \text{ where } 0 \leq r < 4$$

$$\Rightarrow a^3 = (4q + r)^3$$

$$= 64q^3 + r^3 + 12q^2r + 48q^2r$$

$$\Rightarrow a^3 = (64q^3 + 48q^2r + 12q^2r) + r^3 \quad \dots (i)$$

Where, $0 \leq r < 4$



Case I

When $r = 0$,

Putting $r = 0$ in Equation (i), we get

$$a^3 = 64q^3 = 4(16q^3)$$

$$= 4m$$

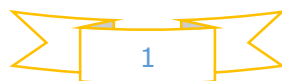
Where $m = 16q^3$ is an integer.

Case II

When $r = 1$,

Putting $r = 1$ in Equation (i), we get

$$a^3 = 64q^3 + 48q^2 + 12q + 1$$



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$$= 4(16q^3 + 12q^2 + 3q) + 1$$

$$= 4m + 1$$

Where $m = (16q^2 + 12q^2 + 3q)$ is an integer.

Case III

When $r = 2$,

Putting $r = 1$ in Equation (i), we get

$$a^3 = 64q^3 + 144q^2 + 108q + 27$$

$$= 64q^3 + 144q^2 + 108q + 24 + 3$$

$$= 4(16q^3 + 36q^2 + 27q + 6) + 3$$

$$= 4m + 3$$

Where $m = (16q^2 + 36q^2 + 27q)$ is an integer.

Hence, the cube of any positive integer is of the form $4m$, $4m + 1$ or $4m + 3$ for some integer m .

