

Mathematics

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(Chapter – 1) (Real Numbers)(Exemplar Problems)
(Class – X)

Exercise 1.3

Question 3:

Show that the square of any positive integer cannot be of the form $5q + 2$ or $5q + 3$ for any integer q .

Answer 3:

Let a be an arbitrary positive integer.

Then, by Euclid's divisions algorithm, corresponding to the positive integers a and 5, there exist non-negative integers m and r such that

$$a = 5m + r, \quad \text{where } 0 \leq r < 5$$

$$\Rightarrow a^2 = (5m + r)^2 = 25m^2 + r^2 + 10mr$$

$$\Rightarrow a^2 = 5(5m^2 + 2mr) + r^2 \quad \dots (1)$$

Where, $0 \leq r < 5$



Case I

When $r = 0$,

Putting $r = 0$ in Equation (i) we get

$$a^2 = 5(5m^2) = 5q$$

Where, $q = 5m^2$ is an integer.

Case II

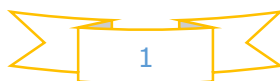
When $r = 1$,

Putting $r = 1$ in Equation (i) we get

$$a^2 = 5(5m^2 + 2m) + 1$$

$$\Rightarrow q = 5q + 1$$

Where, $q = (5m^2 + 2m)$ is an integer.



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Case III

When $r = 2$,

Putting $r = 2$ in Equation (i) we get

$$a^2 = 5(5m^2 + 4m) + 4$$

Where, $q = (5m^2 + 4m)$ is an integer.

Case IV

When $r = 3$,

Putting $r = 3$ in Equation (i) we get

$$a^2 = 5(5m^2 + 6m) + 9$$

$$= 5(5m^2 + 6m) + 5 + 4$$

$$= 5(5m^2 + 6m + 1) + 4 = 5q + 4$$

$$\Rightarrow a = 5q + 1$$

Where, $q = (5m^2 + 6m + 1)$ is an integer.

Case V

When $r = 4$,

Putting $r = 4$ in Equation (i) we get

$$a^2 = 5(5m^2 + 8m) + 16$$

$$= 5(5m^2 + 8m) + 15 + 1$$

$$a^2 = 5(5m^2 + 8m + 3) + 1$$

$$= 5q + 1$$

$$\Rightarrow a = 5q + 1$$

Where, $q = (5m^2 + 8m + 3)$ is an integer.

Hence, the square of any positive integer cannot be of the form $5q + 2$ or $5q + 3$ for any integer q .

