

# Mathematics

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(Chapter – 1) (Real Numbers)(Exemplar Problems)  
(Class – X)

## Exercise 1.3

### Question 4:

Show that the square of any positive integer cannot be of the form  $6m + 2$  or  $6m + 5$  for any integer  $m$ .

### Answer 4:

Let  $a$  be an arbitrary positive integer, then by Euclid's division algorithm, corresponding to the positive integers  $a$  and  $6$ , there exist non – negative integers  $q$  and  $r$  such that

$$\begin{aligned} a &= 6q + r, \quad \text{where } 0 \leq r < 6 \\ \Rightarrow a^2 &= (6q + r)^2 = 36q^2 + r^2 + 12qr \\ \Rightarrow a^2 &= 6(6q^2 + 2qr) + r^2 \end{aligned} \quad \dots (1)$$

Where,  $0 \leq r < 6$



### Case I

When  $r = 0$ ,

Putting  $r = 0$  in Equation (i), we get

$$a^2 = 6(6q)^2 = 6m$$

Where,  $m = 6q^2$  is an Integer.

### Case II

When  $r = 1$ ,

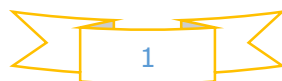
Putting  $r = 1$  in Equation (i), we get

$$a^2 = 6(6q^2 + 2q) + 1$$

$$= 6m + 1$$

$$\Rightarrow a^2 = 6m + 1$$

Where,  $m = (6q^2 + 2q)$  is an Integer.



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## Case III

When  $r = 2$ ,

Putting  $r = 2$  in Equation (i), we get

$$a^2 = 6(6q^2 + 4q) + 4$$

$$= 6m + 4$$

$$\Rightarrow a^2 = 6m + 4$$

Where,  $m = (6q^2 + 4q)$  is an Integer.

## Case IV

When  $r = 3$ ,

Putting  $r = 3$  in Equation (i), we get

$$a^2 = 6(6q^2 + 6q) + 9$$

$$= 6(6q^2 + 6q) + 6 + 3$$

$$= 6(6q^2 + 6q + 1) + 3$$

$$= 6m + 3$$

$$\Rightarrow a^2 = 6m + 3$$

Where,  $m = (6q^2 + 6q + 1)$  is an Integer.

## Case V

When  $r = 4$ ,

Putting  $r = 4$  in Equation (i), we get

$$a^2 = 6(6q^2 + 8q) + 16$$

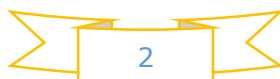
$$= 6(6q^2 + 8q) + 12 + 4$$

$$= 6(6q^2 + 8q + 2) + 4$$

$$= 6m + 4$$

$$\Rightarrow a^2 = 6m + 4$$

Where,  $m = (6q^2 + 8q + 2)$  is an Integer.



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## Case VI

When  $r = 5$ ,

Putting  $r = 5$  in Equation (i), we get

$$\begin{aligned}a^2 &= 6(6q^2 + 10q) + 25 \\&= 6(6q^2 + 10q) + 24 + 1 \\&= 6(6q^2 + 10q + 4) + 1 \\&= 6m + 1 \\&\Rightarrow a^2 = 6m + 1\end{aligned}$$

Where,  $m = (6q^2 + 10q + 4)$  is an Integer.

Hence, the square of any positive integer cannot be of the form  $6m + 2$  or  $6m + 5$  for any integer  $m$ .

