Exercise 1.3

Question 5:

Show that the square of any odd integer is of the form $4m + 1$, for some integer m.

Answer 5:

By Euclid's division algorithm,

we have $a = bq + r$, where $0 \leq r < 4$

Putting $b = 4$, we get

$a = 4q + r$, where $0 \leq r < 4$ i.e. r = 0, 1, 2, 3.

If $r = 0$

$a = 4q$, which is divisible by 2

$\Longrightarrow 4q$ is even.

If $r = 1$

$a = 4q + 1$, which is not divisible by 2.

If $r = 2$

$a = 4q + 2 = 2(2q + 1)$ which is divisible by 2

$\Longrightarrow$ 2(2q+1) is even.

If $r = 3$

$a = 4q + 3$ which is not divisible by 2.

So, for any positive integer q, 4q + 1 and 4q + 3 are odd integers.

Now,

$a^2 = (4q + 1)^2 = 16q^2 + 1 + 8q = 4(4q^2 + 2q) + 1$

$[\because (a + b)^2 = a^2 + 2ab + b^2]$

Which is of the form 4m + 1, where m = $(4q^2 + 2q)$ is an integer.

Now,

$a^2 = (4q + 3)^2 = 16q^2 + 9 + 24q = 4(4q^2 + 6q + 2) + 1$

Which is of the form 4m + 1, where m = $(4q^2 + 6q + 2)$ is an integer.

Hence, for some integer m, the square of any odd integer is of the form 4m + 1.