

# Mathematics

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(Chapter – 1) (Real Numbers)(Exemplar Problems)  
(Class – X)

## Exercise 1.4

### Question 1:

Show that the cube of positive integer of the form  $6q + r$ , where  $q$  is an integer and  $r = 0, 1, 2, 3, 4, 5$  is also of the form  $6m + r$ .

### Answer 1:

Let be  $a$  an arbitrary positive integer. Then by Euclid's division algorithm, corresponding to the positive integers ' $a$ ' and 6, there exist non-negative integers  $q$  and  $r$  such that

$$a = 6q + r, \quad \text{Where } 0 \leq r < 6$$

$$\Rightarrow a^3 = (6q + r)^3 = 216q^3 + r^3 + 3.6q.r(6q + r)$$

$$[\because (a + b)^3 = a^3 + b^3 + 3ab(a + b)]$$

$$\Rightarrow a^3 = (216q^3 + 108q^2r + 18qr^2) + r^3 \quad \dots (i)$$

Where,  $0 \leq r < 6$



### Case I

When  $r = 0$ , putting  $r = 0$  in Equation (i), we get

$$a^3 = (216q^3) = 6(36q^3) = 6m$$

Where,  $m = 36q^3$  is an integer.

### Case II

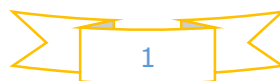
When  $r = 1$ , putting  $r = 1$  in Equation (i), we get

$$a^3 = (216q^3 + 108q^2 + 18q) + 1$$

$$= 6(36q^3 + 18q^2 + 3q) + 1$$

$$\Rightarrow a^3 = 6m + 1,$$

Where  $m = (36q^3 + 18q^2 + 3q)$  is an integer.



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## Case III

When  $r = 2$ , putting  $r = 2$  in Equation (i), we get

$$a^3 = (216q^3 + 216q^2 + 72q) + 8$$

$$a^3 = (216q^3 + 216q^2 + 72q + 6) + 2$$

$$\Rightarrow a^3 = 6(36q^3 + 36q^2 + 12q + 1) + 2 = 6m + 2$$

Where  $m = (36q^3 + 36q^2 + 12q + 1)$  is an integer.

## Case IV

When  $r = 3$ , putting  $r = 3$  in Equation (i), we get

$$a^3 = (216q^3 + 324q^2 + 162q) + 27$$

$$= (216q^3 + 324q^2 + 162q + 24) + 3$$

$$= 6(36q^3 + 54q^2 + 27q + 4) + 3 = 6m + 3$$

Where  $m = (36q^3 + 54q^2 + 27q + 4)$  is an integer.

## Case V

When  $r = 4$ , putting  $r = 4$  in Equation (i), we get

$$a^3 = (216q^3 + 432q^2 + 288q) + 64$$

$$= 6(36q^3 + 72q^2 + 48q) + 60 + 4$$

$$= 6(36q^3 + 72q^2 + 48q + 10) + 4 = 6m + 4$$

Where  $m = (36q^3 + 72q^2 + 48q + 10)$  is an integer.

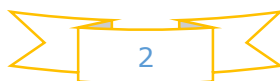
## Case VI

When  $r = 5$ , putting  $r = 5$  in Equation (i), we get

$$a^3 = (216q^3 + 540q^2 + 450q) + 125$$

$$\Rightarrow a^3 = (216q^3 + 540q^2 + 450q + 120) + 5$$

$$\Rightarrow a^3 = 6(36q^3 + 90q^2 + 75q + 20) + 5$$



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$$\Rightarrow a^3 = 6m + 2$$

Where  $m = (36q^3 + 90q^2 + 75q + 20)$  is an integer.

Hence, the cube of a positive integer of the form  $6q + r$ ,  $q$  is an integer and  $r = 0, 1, 2, 3, 4, 5$  is also of the forms  $6m, 6m + 1, 6m + 2, 6m + 3, 6m + 4$  and  $6m + 5$  i.e.,  $6m + r$ .

