Exercise 2.2

Question 1:

Answer the following and justify.

(i) Can $x^2 - 1$ be the quotient on division of $x^6 + 2x^3 + x - 1$ by a polynomial in x in degree 5?

(ii) What will the quotient and remainder be on division of $ax^2 + bx + c$ by $px^3 + qx^2 + rx + s,\ p \neq 0$?

(iii) If on division of polynomial p(x) by a polynomial g(x), the quotient is zero, what is the relation between the degree of p(x) and g(x)?

(iv) If on division of a non-zero polynomial p(x) by polynomial g(x), the remainder is zero, what is the relation between the degrees of p(x) and g(x)?

(v) Can the quadratic polynomial $x^2 + kx + k$ have equal zeroes for some odd integers k > 1?

Answer 1:

(i) No

Because whenever we divide a polynomial $x^6 + 2x^3 + x - 1$ by a polynomial in degree 2, then we get quotient a polynomial in degree 4.

By division algorithm for polynomials,

Dividend = divisor × quotient + remainder

Sum of the degrees of divisor and quotient must be equal to degree of dividend.

(ii) Given that divisor $= px^3 + qx^2 + rx + s,\ p \neq 0$

and dividend $= ax^2 + bx + c$

If degree of dividend < degree of divisor, the quotient will be zero and remainder as same as dividend.

(iii) If division of a polynomial p(x) by a polynomial g(x), the quotient is zero, then relation between the degrees of p(x) and g(x) is degree of p(x)< degree of g(x).

(iv) If division of a non-zero polynomial p(x) by a polynomial g(x), the remainder is zero, then g(x) is a factor of p(x) and has degree less than or equal to the degree of p(x), i.e., degree of g(x) ≤ degree of p(x).

(v) No

Let $p(x) = x^2 + kx + k$

Let α and α be the zeroes of the polynomial $p(x)$.

We know that,

∴ sum of zeroes $\alpha + \alpha = -\frac{b}{a}$

$\Rightarrow 2\alpha = -\frac{k}{1} = -k$

$\Rightarrow \alpha = -\frac{k}{2}$ … (i)

and product of zeroes $\alpha.\alpha = \frac{c}{a}$

$\Rightarrow \alpha^2 = \frac{k}{1} = k$ … (ii)

Solving equations (i) and (ii), we get

$\frac{k^2}{4} = k$

$\Rightarrow k^2 = 4k$

$\Rightarrow k^2 - 4k = 0$

$\Rightarrow k(k-4) = 0$

$\Rightarrow k = 0$ or $k = 4$

But k > 1, so $k = 4$ which is even not odd number.