Exercise 2.2

Question 2:

Are the following statements 'True' or 'False'? Justify your answer.

(i) If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both positive, then a, b and c all have the same sign.

(ii) If the graph of polynomial intersects the x-axis at only one point, it cannot be quadratic polynomial.

(iii) If the graph of a polynomial intersects the x-axis at exactly two points, it need not be a quadratic polynomial.

(iv) If two of the zeroes of a cubic polynomial are zero, then it does not have linear and constant terms.

(v) If all the zeroes of a cubic polynomial are negative, then all the coefficients and the constant term of the polynomial have the same sign.

(vi) If all the zeroes of a cubic polynomial $x^3 + ax^2 - bx + c$ are positive, then at least one of a, b and c is non-negative.

(vii) The only value of k for which the quadratic polynomial $kx^2 + x + k$ has equal zeroes is $\frac{1}{2}$.

Answer 2:

(i) False

If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both positive then

$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha.\beta = \frac{c}{a}$$

Where α and β are the zeroes of quadratic polynomial.

If $c > 0$ and $a > 0$ then $b < 0$ or if $c < 0$ and $a < 0$ then $b > 0$

(ii) True

If the graph of a polynomial intersects the x – axis at only one point, then it cannot be a quadratic polynomial because a quadratic polynomial may touch the x-axis at exactly one point or intersects x-axis at exactly two points or do not touch the x-axis.

(iii) True

If the graph of a polynomial intersects the x-axis at exactly two point, then it may or may not be a quadratic polynomial. As, a polynomial of degree more than 2 is possible which intersects the x-axis at exactly two points when it has two real roots and other imaginary roots.

(iv) True

Let α, β and γ be the zeroes of the cubic polynomial and given that two of the zeroes have value 0.

Let $\beta = \gamma = 0$

And $f(x) = (x-\alpha)(x-\beta)(x-\gamma)$

$= (x-\alpha)(x-0)(x-0)$

$= x^3 - ax^2$

Which does not have linear and constant terms.

(v) True

If $f(x) = ax^3 + bx^2 + cx + d$. then, for all negative roots, a, b, c and d must have same sign.

(vi) False

Let α, β and γ be the three zeroes of cubic polynomial $x^3 + ax^2 - bx + c$

Then, product of zeroes = $-\frac{Constant\ term}{coefficinet\ of\ x^3}$

$$\Rightarrow \alpha\beta\gamma = -\frac{c}{1}$$

$\Rightarrow \alpha\beta\gamma = -c$

Given that, all three zeroes are positive. So, the product of all three zeroes should also be positive.

So, $\alpha\beta\gamma > 0$

$\Rightarrow -c > 0$

$\Rightarrow c < 0$

Now, sum of the zeroes = $\alpha + \beta + \gamma = -\frac{coefficinet\ of\ x^2}{coefficinet\ of\ x^3}$

$\Rightarrow \alpha + \beta + \gamma = -\frac{a}{1} = -a$

But α, β and γ are all positive.

So, $\alpha + \beta + \gamma > 0$

$\Rightarrow -a > 0$

$\Rightarrow a < 0$

Sum of the product of two zeroes at a time = $\frac{Ccoefficinet\ of\ x}{coefficinet\ of\ x^3}$

$\Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = \frac{-b}{1}$

$\because \alpha\beta + \beta\gamma + \alpha\gamma > 0 \qquad \Rightarrow -b > 0$

$\Rightarrow b < 0$

So, the cubic polynomial $x^3 + ax^2 - bx + c$ has all three zeroes which are positive only when all constants a, b and c are negative.

(vii) False

Let $f(x) = kx^2 + x + k$

Let α and α be the zeroes of the polynomial $p(x)$.

We know that,

$\therefore$ sum of zeroes $\alpha + \alpha = -\frac{b}{a}$

$\Rightarrow 2\alpha = -\frac{1}{k}$

$\Rightarrow \alpha = -\frac{1}{2k}$ … (i)

and product of zeroes $\alpha.\alpha = \frac{c}{a}$

$\Rightarrow \alpha^2 = \frac{k}{k} = 1$ … (ii)

Solving equations (i) and (ii), we get

$\frac{1}{4k^2} = 1$

$\Rightarrow 4k^2 = 1$

$\Rightarrow k^2 = \frac{1}{4}$

$\Rightarrow k = \pm\frac{1}{2}$

$\Rightarrow k = \frac{1}{2}$ or $k = -\frac{1}{2}$

So, for two values of k, given quadratic polynomial has equal zeroes.