

Mathematics

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(Chapter – 2) (Polynomials)(Exemplar Problems)

(Class – X)

Exercise 2.2

Question 2:

Are the following statements ‘True’ or ‘False’? Justify your answer.

- (i) If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both positive, then a , b and c all have the same sign.
- (ii) If the graph of polynomial intersects the x-axis at only one point, it cannot be quadratic polynomial.
- (iii) If the graph of a polynomial intersects the x-axis at exactly two points, it need not be a quadratic polynomial.
- (iv) If two of the zeroes of a cubic polynomial are zero, then it does not have linear and constant terms.
- (v) If all the zeroes of a cubic polynomial are negative, then all the coefficients and the constant term of the polynomial have the same sign.
- (vi) If all the zeroes of a cubic polynomial $x^3 + ax^2 - bx + c$ are positive, then at least one of a , b and c is non-negative.
- (vii) The only value of k for which the quadratic polynomial $kx^2 + x + k$ has equal zeroes is $\frac{1}{2}$.

Answer 2:

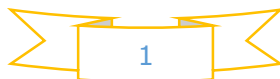
(i) False

If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both positive then

$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha \cdot \beta = \frac{c}{a}$$

Where α and β are the zeroes of quadratic polynomial.

If $c > 0$ and $a > 0$ then $b < 0$ or if $c < 0$ and $a < 0$ then $b > 0$



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(ii) True

If the graph of a polynomial intersects the x – axis at only one point, then it cannot be a quadratic polynomial because a quadratic polynomial may touch the x-axis at exactly one point or intersects x-axis at exactly two points or do not touch the x-axis.

(iii) True

If the graph of a polynomial intersects the x-axis at exactly two point, then it may or may not be a quadratic polynomial. As, a polynomial of degree more than 2 is possible which intersects the x-axis at exactly two points when it has two real roots and other imaginary roots.

(iv) True

Let α, β and γ be the zeroes of the cubic polynomial and given that two of the zeroes have value 0.

Let $\beta = \gamma = 0$

And $f(x) = (x - \alpha)(x - \beta)(x - \gamma)$

$= (x - \alpha)(x - 0)(x - 0)$

$= x^3 - \alpha x^2$

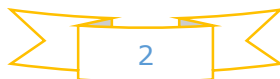
Which does not have linear and constant terms.

(v) True

If $f(x) = ax^3 + bx^2 + cx + d$. then, for all negative roots, a, b, c and d must have same sign.

(vi) False

Let α, β and γ be the three zeroes of cubic polynomial $x^3 + ax^2 - bx + c$



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Then, product of zeroes = $-\frac{\text{Constant term}}{\text{coefficinet of } x^3}$

$$\Rightarrow \alpha\beta\gamma = -\frac{c}{1}$$

$$\Rightarrow \alpha\beta\gamma = -c$$

Given that, all three zeroes are positive. So, the product of all three zeroes should also be positive.

$$\text{So, } \alpha\beta\gamma > 0$$

$$\Rightarrow -c > 0$$

$$\Rightarrow c < 0$$

Now, sum of the zeroes = $\alpha + \beta + \gamma = -\frac{\text{coefficinet of } x^2}{\text{coefficinet of } x^3}$

$$\Rightarrow \alpha + \beta + \gamma = -\frac{a}{1} = -a$$

But α, β and γ are all positive.

$$\text{So, } \alpha + \beta + \gamma > 0$$

$$\Rightarrow -a > 0$$

$$\Rightarrow a < 0$$

Sum of the product of two zeroes at a time = $\frac{\text{Ccoefficinet of } x}{\text{coefficinet of } x^3}$

$$\Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = \frac{-b}{1}$$

$$\therefore \alpha\beta + \beta\gamma + \alpha\gamma > 0 \quad \Rightarrow -b > 0$$

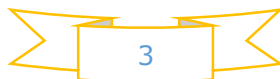
$$\Rightarrow b < 0$$

So, the cubic polynomial $x^3 + ax^2 - bx + c$ has all three zeroes which are positive only when all constants a, b and c are negative.

(vii) False

$$\text{Let } f(x) = kx^2 + x + k$$

Let α and α be the zeroes of the polynomial $p(x)$.



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We know that,

$$\therefore \text{sum of zeroes } \alpha + \alpha = -\frac{b}{a}$$

$$\Rightarrow 2\alpha = -\frac{1}{k}$$

$$\Rightarrow \alpha = -\frac{1}{2k} \quad \dots \text{ (i)}$$

and product of zeroes $\alpha \cdot \alpha = \frac{c}{a}$

$$\Rightarrow \alpha^2 = \frac{k}{k} = 1 \quad \dots \text{ (ii)}$$

Solving equations (i) and (ii), we get

$$\frac{1}{4k^2} = 1$$

$$\Rightarrow 4k^2 = 1$$

$$\Rightarrow k^2 = \frac{1}{4}$$

$$\Rightarrow k = \pm \frac{1}{2}$$

$$\Rightarrow k = \frac{1}{2} \text{ or } k = -\frac{1}{2}$$

So, for two values of k, given quadratic polynomial has equal zeroes.

