Exercise 2.4

Question 4:

Find k, so that $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Also, find all the zeroes of the two polynomials.

Answer 4:

Given that $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$, then applying division algorithm,

$$
\begin{array}{r|l}
 & 2x^2 - 3x - (2k+8) \\
x^2 + 2x + k & 2x^4 + x^3 - 14x^2 + 5x + 6 \\
 & \underline{-\,2x^4 \pm 4x^3 \pm 2kx^2} \\
 & \quad -3x^3 - (2k+14)x^2 + 5x + 6 \\
 & \quad \underline{\mp 3x^3 \mp 6x^2 \mp 3kx} \\
 & \qquad -(2k+8)x^2 + (5+3k)x + 6 \\
 & \qquad \underline{\mp (2k+8)x^2 \mp (16+4k)x \mp (2k^2+8k)} \\
 & \qquad\qquad (21+7k)x + (2k^2+8k+6)
\end{array}
$$

Since, $(x^2 + 2x + k)$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$.

So, when we apply division algorithm remainder should be zero.

$\therefore (7k + 21) = 0$ and $(2k^2 + 8k + 6) = 0$

$\Rightarrow 7k + 21 = 0$ and $2k^2 + 8k + 6 = 0$

$\Rightarrow k = -3$ and $k^2 + 4k + 3 = 0$

$\Rightarrow k = -3$ and $k^2 + 3k + k + 3 = 0$

$\Rightarrow k = -3$ and $k(k + 3) + 1(k + 3) = 0$

$\Rightarrow k = -3$ and $(k + 1)(k + 3) = 0$

$\Longrightarrow k = -3$ and $k = -1$ or -3

Here, only $k = -3$ satisfy the required condition.

Thus, the required value of k is – 3.

Now, Dividend = Divisor × Quotient + Remainder

$\Longrightarrow 2x^4 + x^3 - 14x^2 + 5x + 6 = (x^2 + 2x - 3)(2x^2 - 3x - 2) + 0$

Using factorization method,

$(x^2 + 3x - x - 3)(2x^2 - 4x + x - 2)$

$= \{x(x + 3) - 1(x + 3)\}\{2x(x - 2) + 1(x - 2)\}$

$= (x - 1)(x + 3(x - 2)(2x + 1)$

Hence, the zeroes of $x^2 + 2x - 3$ are 1, – 3 and the zeroes of $2x^4 + x^3 - 14x^2 + 5x + 6$ are 1, – 3, 2, $-\frac{1}{2}$.