Exercise 2.4

Question 5:

If $x-\sqrt{5}$ is a factor of the cubic polynomial $x^3-3\sqrt{5}x^2+13x-3\sqrt{5}$, then find all the zeroes of the polynomial.

Answer 5:

Let $f(x)=x^3-3\sqrt{5}x^2+13x-3\sqrt{5}$ and given that, $(x-\sqrt{5})$ is a one of the factor of $f(x)$.

Now, using division algorithm.

$$
\begin{array}{r|l}
 & x^2-2\sqrt{5}x+3 \\
\hline
x-\sqrt{5} & x^3-3\sqrt{5}x^2+13x-3\sqrt{5} \\
 & \underline{{}_{-}x^3 \mp \sqrt{5}x^2} \\
 & \quad -2\sqrt{5}x^2+13x-3\sqrt{5} \\
 & \quad \underline{\mp 2\sqrt{5}x^2 \pm 10x} \\
 & \qquad\qquad 13x-3\sqrt{5} \\
 & \qquad\qquad \underline{{}_{-}13x \mp 3\sqrt{5}} \\
 & \qquad\qquad\qquad 0
\end{array}
$$

$\therefore x^3-3\sqrt{5}x^2+13x-3\sqrt{5}=(x^2-2\sqrt{5}x+3)\times(x-\sqrt{5})+0$

[$\because$ Dividend = divisor × quotient + remainder]

$=(x-\sqrt{5})[x^2-\{(\sqrt{5}+\sqrt{2})+(\sqrt{5}-\sqrt{2}\}x+3]$

$=(x-\sqrt{5})[x^2-(\sqrt{5}-\sqrt{2})x-(\sqrt{5}-\sqrt{2})x+(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})]$

[$\because 3=(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})$]

$= (x - \sqrt{5})[x\{x - (\sqrt{5} + \sqrt{2})\} - (\sqrt{5} - \sqrt{2})\{x - (\sqrt{5} + \sqrt{2})\}]$

$= (x - \sqrt{5})\{x - (\sqrt{5} + \sqrt{2})\}\{x - (\sqrt{5} - \sqrt{2})\}$

Hence, all the zeroes of polynomial are $\sqrt{5}$, $(\sqrt{5} + \sqrt{2})$ and $(\sqrt{5} - \sqrt{2})$.