Exercise 2.4

Question 6:

For which value of a and b, are zeroes of $q(x) = x^3 + 2x^2 + a$ the zeroes of polynomial $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$? which zeroes of $p(x)$ are not the zeroes of $q(x)$?

Answer 6:

Given that the zeroes of $q(x) = x^3 + 2x^2 + a$ are also the zeroes of the polynomial $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$ i.e., $q(x)$ is a factor of $p(x)$. Using division algorithm.

$$x^2 - 3x + 2$$

$$x^3 + 2x^2 + a \,\big)\, x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$$

$$\underline{_{-}\, x^5 \pm 2x^4 \qquad \pm ax^2}$$

$$-3x^4 - 4x^3 - (a-3)x^2 + 3x + b$$

$$\underline{\mp 3x^4 \mp 6x^3 \qquad \mp 3ax}$$

$$2x^3 - (a-3)x^2 + (3+3a)x + b$$

$$\underline{_{-}\, 2x^3 \pm 4x^2 \qquad \pm 2a}$$

$$-(a+1)x^2 + (3+3a)x + b - 2a$$

If $(x^3 + 2x^2 + a)$ is a factor of $x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$, then remainder should be zero.

i.e., $-(1+a)x^2 + (3+3a)x + (b-2a) = 0.x^2 + 0.x + 0$

On comparing the coefficient of x^2 and constant term, we get

$a + 1 = 0$

$\Rightarrow a = -1$

and $b - 2a = 0$

$\Longrightarrow b = 2a$

$\Longrightarrow b = 2(-1) = -2$ $[\because a = -1]$

For $a = -1$ and $b = -2$, the zeroes of $q(x)$ are also the zeroes of the polynomial $p(x)$.

$\therefore q(x) = x^3 + 2x^2 - 1$

And $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x - 2$

Now, Dividend = divisor × quotient + remainder

$p(x) = (x^3 + 2x^2 - 1)(x^2 - 3x + 2) + 0$

$= (x^3 + 2x^2 - 1)\{x^2 - 2x - 2 + x\}$

$= (x^3 + 2x^2 - 1)(x - 2)(x - 1)$

Hence, the zeroes of $p(x)$ are 1 and 2 which are not the zeroes of $q(x)$.