(Exemplar Problems)

Short Answer (S.A.)

Question 10:

Show that $cos\left(2tan^{-1}\frac{1}{7}\right) = sin\left(4tan^{-1}\frac{1}{3}\right)$.

Answer 10:

LHS

$$= cos\left(2tan^{-1}\frac{1}{7}\right)$$

$$= cos\left[cos^{-1}\frac{1-\left(\frac{1}{7}\right)^2}{1+\left(\frac{1}{7}\right)^2}\right]$$

$$\left[as\ 2tan^{-1}x = cos^{-1}\frac{1-x^2}{1+x^2}\right]$$

$$= cos\left[cos^{-1}\frac{48/49}{50/49}\right]$$

$$= cos\left[cos^{-1}\frac{48}{50}\right] = \frac{48}{50} = \frac{24}{25}$$

Now, RHS

$$= sin\left(4tan^{-1}\frac{1}{3}\right)$$

$$= sin\left[2.\left(2tan^{-1}\frac{1}{3}\right)\right]$$

$$= sin\left[2tan^{-1}\frac{2\left(\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)^2}\right]$$

$$\left[as\ 2tan^{-1}x = tan^{-1}\frac{2x}{1-x^2}\right]$$

$$= sin\left[2tan^{-1}\frac{2/3}{8/9}\right]$$

$$= sin\left[2tan^{-1}\frac{3}{4}\right]$$

$$= sin\left[sin^{-1}\frac{2\left(\frac{3}{4}\right)}{1+\left(\frac{3}{4}\right)^2}\right]$$

$$\left[as\ 2tan^{-1}x = sin^{-1}\frac{2x}{1+x^2}\right]$$

$$= sin\left[sin^{-1}\frac{3/2}{25/16}\right]$$

$$= sin\left[sin^{-1}\frac{48}{50}\right] = \frac{48}{50} = \frac{24}{25}$$

Hence, LHS = RHS = $\frac{24}{25}$