

Mathematics

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(Chapter 2)(Inverse Trigonometric Functions)

(Class XII)

(Exemplar Problems)

Short Answer (S.A.)

Question 10:

Show that $\cos\left(2\tan^{-1}\frac{1}{7}\right) = \sin\left(4\tan^{-1}\frac{1}{3}\right)$.

Answer 10:

LHS

$$= \cos\left(2\tan^{-1}\frac{1}{7}\right)$$

$$= \cos\left[\cos^{-1}\frac{1 - \left(\frac{1}{7}\right)^2}{1 + \left(\frac{1}{7}\right)^2}\right]$$



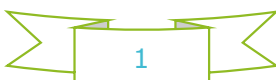
$$\left[as\ 2\tan^{-1}x = \cos^{-1}\frac{1 - x^2}{1 + x^2}\right]$$

$$= \cos\left[\cos^{-1}\frac{48/49}{50/49}\right]$$

$$= \cos\left[\cos^{-1}\frac{48}{50}\right] = \frac{48}{50} = \frac{24}{25}$$

Now, RHS

$$= \sin\left(4\tan^{-1}\frac{1}{3}\right)$$



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$$= \sin \left[2 \cdot \left(2 \tan^{-1} \frac{1}{3} \right) \right]$$

$$= \sin \left[2 \tan^{-1} \frac{2 \left(\frac{1}{3} \right)}{1 - \left(\frac{1}{3} \right)^2} \right]$$

$$\left[\text{as } 2 \tan^{-1} x = \tan^{-1} \frac{2x}{1 - x^2} \right]$$

$$= \sin \left[2 \tan^{-1} \frac{2/3}{8/9} \right]$$

$$= \sin \left[2 \tan^{-1} \frac{3}{4} \right]$$



$$= \sin \left[\sin^{-1} \frac{2 \left(\frac{3}{4} \right)}{1 + \left(\frac{3}{4} \right)^2} \right]$$

$$\left[\text{as } 2 \tan^{-1} x = \sin^{-1} \frac{2x}{1 + x^2} \right]$$

$$= \sin \left[\sin^{-1} \frac{3/2}{25/16} \right]$$

$$= \sin \left[\sin^{-1} \frac{48}{50} \right] = \frac{48}{50} = \frac{24}{25}$$

$$\text{Hence, LHS} = \text{RHS} = \frac{24}{25}$$

