Mathematics

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(Chapter 2)(Inverse Trigonometric Functions)

(Class XII)

(Exemplar Problems)

Long Answer (L.A.)

Question 14:

Prove that $sin^{-1}\frac{8}{17} + sin^{-1}\frac{3}{5} = sin^{-1}\frac{77}{85}$

Answer 14:

LHS = $sin^{-1}\frac{8}{17} + sin^{-1}\frac{3}{5}$

$$= tan^{-1}\frac{8}{\sqrt{17^2 - 8^2}} + tan^{-1}\frac{3}{\sqrt{5^2 - 3^2}}$$

$$\left[as\ sin^{-1}\frac{a}{b} = tan^{-1}\frac{a}{\sqrt{b^2 - a^2}}\right]$$

$$= tan^{-1}\frac{8}{15} + tan^{-1}\frac{3}{4}$$

$$= tan^{-1}\left[\frac{\frac{8}{15} + \frac{3}{4}}{1 - \frac{8}{15} \times \frac{3}{4}}\right]$$

$$\left[as\ tan^{-1}x + tan^{-1}y = tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$= tan^{-1}\left[\frac{\frac{32 + 45}{15 \times 4}}{\frac{15 \times 4 - 8 \times 3}{15 \times 4}}\right]$$

$$= tan^{-1}\left[\frac{\frac{77}{60}}{\frac{36}{60}}\right]$$

$$= tan^{-1}\frac{77}{36}$$

$$= sin^{-1}\frac{77}{\sqrt{77^2 + 36^2}}$$

$$\left[as\ tan^{-1}\frac{a}{b} = cos^{-1}\frac{b}{\sqrt{a^2 + b^2}}\right]$$

$$= sin^{-1}\frac{77}{\sqrt{5929 + 1296}}$$

$$= sin^{-1}\frac{77}{\sqrt{5929 + 1296}}$$

$$= sin^{-1}\frac{77}{\sqrt{7225}}$$

$$= sin^{-1}\frac{77}{85}$$

$$= \text{RHS}$$