Mathematics

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(Chapter 2)(Inverse Trigonometric Functions)

(Class XII)

(Exemplar Problems)

Long Answer (L.A.)

Question 15:

Show that $sin^{-1}\frac{5}{13} + cos^{-1}\frac{3}{5} = tan^{-1}\frac{63}{16}$.

Answer 15:

LHS $= sin^{-1}\frac{5}{13} + cos^{-1}\frac{3}{5}$

$$= tan^{-1}\frac{5}{\sqrt{13^2 - 5^2}} + tan^{-1}\frac{\sqrt{5^2 - 3^2}}{3}$$

$$\left[as\ sin^{-1}\frac{a}{b} = tan^{-1}\frac{a}{\sqrt{b^2 - a^2}} \text{ and } cos^{-1}\frac{a}{b} = tan^{-1}\frac{\sqrt{b^2 - a^2}}{a}\right]$$

$$= tan^{-1}\frac{5}{12} + tan^{-1}\frac{4}{3}$$

$$= tan^{-1}\left[\frac{\frac{5}{12} + \frac{4}{3}}{1 - \frac{5}{12} \times \frac{4}{3}}\right]$$

$$\left[as\ tan^{-1}x + tan^{-1}y = tan^{-1}\left(\frac{x + y}{1 - xy}\right)\right]$$

$$= tan^{-1}\left[\frac{\frac{15+48}{12\times 3}}{\frac{12\times 3-5\times 4}{12\times 3}}\right]$$

$$= tan^{-1}\left[\frac{\frac{63}{36}}{\frac{16}{36}}\right]$$

$$= tan^{-1}\frac{63}{16}$$

$$= \text{RHS}$$