(Exemplar Problems)
Long Answer (L.A.)

Question 19:

If $a_1, a_2, a_3, \dots, a_n$ is an arithmetic progression with common difference d, then evaluate the following expression

$$tan\left[tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+tan^{-1}\left(\frac{d}{1+a_3a_4}\right)+\cdots+tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right]$$

Answer 19:

Given that:

$$tan\left[tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+tan^{-1}\left(\frac{d}{1+a_3a_4}\right)+\cdots+tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right]$$

$$= tan\left[tan^{-1}\left(\frac{a_2-a_1}{1+a_1a_2}\right)+tan^{-1}\left(\frac{a_3-a_2}{1+a_2a_3}\right)+tan^{-1}\left(\frac{a_4-a_3}{1+a_3a_4}\right)+\cdots+tan^{-1}\left(\frac{a_n-a_{n-1}}{1+a_{n-1}a_n}\right)\right]$$

[d $= a_2 - a_1 = a_3 - a_2 = a_4 - a_3.$ as $a_1, a_2, a_3, a_4, \dots$ are in AP]

$$= tan[(tan^{-1}a_2 - tan^{-1}a_1) + (tan^{-1}a_3 - tan^{-1}a_2) + (tan^{-1}a_4 - tan^{-1}a_3) + \cdots + (tan^{-1}a_n - tan^{-1}a_{n-1})]$$

$$\left[as\ tan^{-1}\left(\frac{x-y}{1+xy}\right) = tan^{-1}x - tan^{-1}y\right]$$

$$= tan[(tan^{-1}a_n - tan^{-1}a_1)]$$

$$= tan\left[tan^{-1}\left(\frac{a_n-a_1}{1+a_na_1}\right)\right]$$

$$\left[as\ tan^{-1}x - tan^{-1}y = tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$$

$$= \frac{a_n - a_1}{1 + a_n a_1}$$