(Exemplar Problems)

Short Answer (S.A.)

Question 3:

Prove that $cot\left(\frac{\pi}{4} - 2cot^{-1}3\right) = 7$

Answer 3:

LHS = $cot\left(\frac{\pi}{4} - 2cot^{-1}3\right)$

$= cot\left(\frac{\pi}{4} - 2tan^{-1}\frac{1}{3}\right)$

$= cot\left(\frac{\pi}{4} - tan^{-1}\left[\frac{2\times\frac{1}{3}}{1-\left(\frac{1}{3}\right)^2}\right]\right)$ $\quad [\, as\ 2tan^{-1}x = tan^{-1}\frac{2x}{1-x^2}\,]$

$= cot\left(\frac{\pi}{4} - tan^{-1}\frac{3}{4}\right)$

$= cot\left(\frac{\pi}{4} - cot^{-1}\frac{4}{3}\right)$

$= \frac{cot\frac{\pi}{4}cot\left(cot^{-1}\frac{4}{3}\right)+1}{cot\left(cot^{-1}\frac{4}{3}\right)-cot\frac{\pi}{4}}$ $\quad \left[as\ cot(A-B) = \frac{\cot A cot B + 1}{\cot B - \cot A}\right]$

$= \frac{\frac{4}{3}+1}{\frac{4}{3}-1} = \frac{\frac{7}{3}}{\frac{1}{3}} = 7 = RHS$