

Mathematics

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(Chapter 2)(Inverse Trigonometric Functions)

(Class XII)

(Exemplar Problems)

Short Answer (S.A.)

Question 4:

Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.

Answer 4:

$$\begin{aligned} \text{We have } & \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right] \\ &= \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \tan^{-1}(\sqrt{3}) + \tan^{-1}\left[-\sin\left(\frac{\pi}{2}\right)\right] \\ &= \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \tan^{-1}(\sqrt{3}) + \tan^{-1}[-1] \\ &= \tan^{-1}\left(-\tan\frac{\pi}{6}\right) + \tan^{-1}\left(\tan\frac{\pi}{3}\right) + \tan^{-1}\left[-\tan\frac{\pi}{4}\right] \\ &= \tan^{-1}\left[\tan\left(-\frac{\pi}{6}\right)\right] + \tan^{-1}\left(\tan\frac{\pi}{3}\right) + \tan^{-1}\left[\tan\left(-\frac{\pi}{4}\right)\right] \end{aligned}$$

[As we know that the range of the principal value branch of \tan^{-1} is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$]

$$= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4} = \frac{-2\pi + 4\pi - 3\pi}{12} = -\frac{\pi}{12}$$

Hence, the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right] = -\frac{\pi}{12}$

