

Mathematics

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(Chapter 2)(Inverse Trigonometric Functions)

(Class XII)

(Exemplar Problems)

Short Answer (S.A.)

Question 8:

Find the value of the expression $\sin\left(2\tan^{-1}\frac{1}{3}\right) + \cos(\tan^{-1}2\sqrt{2})$.

Answer 8:

Given that $\sin\left(2\tan^{-1}\frac{1}{3}\right) + \cos(\tan^{-1}2\sqrt{2})$

$$= \sin\left(\tan^{-1}\left(\frac{2 \times \frac{1}{3}}{1 - \left(\frac{1}{3}\right)^2}\right)\right) + \cos(\tan^{-1}2\sqrt{2})$$

$$\left[as\ 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$$

$$= \sin\left[\tan^{-1}\left(\frac{\frac{2}{3}}{\frac{8}{9}}\right)\right] + \cos(\tan^{-1}2\sqrt{2})$$

$$= \sin\left[\tan^{-1}\frac{3}{4}\right] + \cos(\tan^{-1}2\sqrt{2})$$

$$= \sin\left[\sin^{-1}\frac{3}{\sqrt{9+16}}\right] + \cos\left(\cos^{-1}\frac{1}{\sqrt{8+1}}\right)$$

$$\left[\left[as\ \tan^{-1}\frac{a}{b} = \sin^{-1}\frac{a}{\sqrt{a^2+b^2}}\right] \text{ and } \left[as\ \tan^{-1}\frac{a}{b} = \cos^{-1}\frac{b}{\sqrt{a^2+b^2}}\right]\right]$$

$$= \sin\left[\sin^{-1}\frac{3}{5}\right] + \cos\left(\cos^{-1}\frac{1}{3}\right)$$

$$= \frac{3}{5} + \frac{1}{3} = \frac{9+5}{15} = \frac{14}{15}$$

