

Mathematics

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(Chapter – 12) (Heron's Formula)(Exemplar Problems)

(Class – IX)

Exercise 12.3

Question 3:

From a point in the interior of an equilateral triangle, perpendiculars are drawn on the three sides. The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm. Find the area of the triangle.

Answer 3:

Let ABC be an equilateral triangle, O be the interior point and AQ, BR and PC are the perpendicular drawn from point O.

Let sides of an equilateral triangle be a m.

$$\begin{aligned} \text{Area of } \triangle OAB &= \frac{1}{2} \times AB \times OP && [\because \text{area of a triangle} = \frac{1}{2} (\text{base} \times \text{height})] \\ &= \frac{1}{2} \times a \times 14 = 7a \text{ cm}^2 && \dots\dots\dots(i) \end{aligned}$$

$$\begin{aligned} \text{Area of } \triangle OBC &= \frac{1}{2} \times BC \times OQ \\ &= \frac{1}{2} \times a \times 10 = 5a \text{ cm}^2 && \dots\dots\dots(ii) \end{aligned}$$

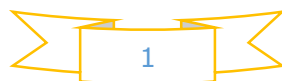
$$\begin{aligned} \text{Area of } \triangle OAC &= \frac{1}{2} \times AC \times OR \\ &= \frac{1}{2} \times a \times 6 = 3a \text{ cm}^2 && \dots\dots\dots(iii) \end{aligned}$$

$$\begin{aligned} \therefore \text{Area of an equilateral } \triangle ABC &= \text{Area of } (\triangle OAB + \triangle OBC + \triangle OAC) \\ &= (7a + 5a + 3a) = 15a \text{ cm}^2 && \dots\dots\dots(iv) \end{aligned}$$

$$\text{We have, semi - perimeter } s = \frac{a+a+a}{2} \Rightarrow s = \frac{3a}{2} \text{ cm}$$

$$\begin{aligned} \therefore \text{Area of an equilateral } \triangle ABC &= \sqrt{s(s-a)(s-b)(s-c)} \\ &&& [\text{by Heron's formula}] \end{aligned}$$

$$= \sqrt{\frac{3a}{2} \left(\frac{3a}{2} - a \right) \left(\frac{3a}{2} - a \right) \left(\frac{3a}{2} - a \right)}$$



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$$= \sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}} = \frac{\sqrt{3}}{4} a^2 \quad \dots\dots\dots(v)$$

From Equations (iv) and (v),

$$\frac{\sqrt{3}}{4} a^2 = 15 a$$

$$\Rightarrow a = \frac{15 \times 4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{60\sqrt{3}}{3} = 20\sqrt{3} \text{ cm}$$

On putting $a = 20\sqrt{3}$ in Equation (v), we get

$$\begin{aligned} \text{Area of a } \Delta ABC &= \frac{\sqrt{3}}{4} (20\sqrt{3})^2 \\ &= \frac{\sqrt{3}}{4} \times 400 \times 3 \\ &= 300\sqrt{3} \text{ cm}^2 \end{aligned}$$

Hence, the area of an equilateral triangle is $300\sqrt{3} \text{ cm}^2$

