Exercise 12.3

Question 3:

From a point in the interior of an equilateral triangle, perpendiculars are drawn on the three sides. The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm. Find the area of the triangle.

Answer 3:

Let ABC be an equilateral triangle, O be the interior point and AQ, BR and PC are the perpendicular drawn from point O.

Let sides of an equilateral triangle be a m.

Area of $\Delta OAB = \frac{1}{2} \times AB \times OP$ [∵ area of a triangle = $\frac{1}{2}$ (base × height)]

$= \frac{1}{2} \times a \times 14 = 7a\ cm^2$ ……………(i)

Area of $\Delta OBC = \frac{1}{2} \times BC \times OQ$

$= \frac{1}{2} \times a \times 10 = 5a\ cm^2$ ……………(ii)

Area of $\Delta OAC = \frac{1}{2} \times AC \times OR$

$= \frac{1}{2} \times a \times 6 = 3a\ cm^2$ ……………(iii)

∴ Area of an equilateralΔABC = Area of (ΔOAB + ΔOBC + ΔOAC)

$= (7a + 5a + 3a) = 15a\ cm^2$ ……………(iv)

We have, semi – perimeter $s = \frac{a+a+a}{2} \Rightarrow s = \frac{3a}{2}\ cm$

∴ Area of an equilateral ΔABC $= \sqrt{s(s-a)(s-b)(s-c)}$

[by Heron's formula]

$$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2} - a\right)\left(\frac{3a}{2} - a\right)\left(\frac{3a}{2} - a\right)}$$

$$= \sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}} = \frac{\sqrt{3}}{4}a^2 \qquad \text{...............(v)}$$

From Equations (iv) and (v),

$$\frac{\sqrt{3}}{4}a^2 = 15\,a$$

$$\Rightarrow \qquad a = \frac{15X4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{60\sqrt{3}}{3} = 20\sqrt{3}\ cm$$

On putting = $20\sqrt{3}$ in Equation (v), we get

Area of a $\Delta ABC = \frac{\sqrt{3}}{4}(20\sqrt{3})^2$

$= \frac{\sqrt{3}}{4} \times 400 \times 3$

$= 300\sqrt{3}cm^2$

Hence, the area of an equilateral triangle is $300\sqrt{3}cm^2$