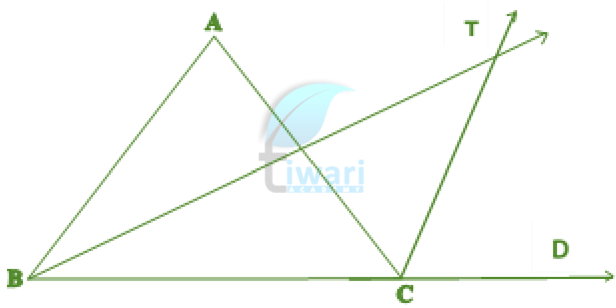
Exercise 6.4

Question 2:
Bisectors of interior ∠B and exterior ∠ACD of a ΔABC intersect at the point T. Prove that ∠BTC = $\frac{1}{2}$ ∠BAC.

Answer 2:
Given:
ΔABC, produce BC to D and the bisectors of $\angle ABC$ and $\angle ACD$ meet at point T.

To Prove:
$$\angle BTC = \frac{1}{2}\angle BAC$$



Proof:
ΔABC, ∠ACD is an exterior angle.

∴ $\angle ACD = \angle ABC + \angle CAB$
[Exterior angle of a triangle is equal to the sum of two opposite angles]

⇒ $\frac{1}{2}\angle ACD = \frac{1}{2}\angle CAB + \frac{1}{2}\angle ABC$
[Dividing both sides by 2]

⇒ $\angle TCD = \frac{1}{2}\angle CAB + \frac{1}{2}\angle ABC$ …(i)
[∵ CT is a bisector of $\angle ACD$ ⇒ $\frac{1}{2}\angle ACD = \angle TCD$]

In ΔBTC,
∴ $\angle TCD = \angle BTC + \angle CBT$
[Exterior angle of a triangle is equal to the sum of two opposite angles]

$\Rightarrow \qquad \angle TCD = \angle BTC + \frac{1}{2}\angle ABC \qquad \ldots$(ii)

[$\because$ BT bisector of $\angle ABC \Rightarrow \angle CBT = \frac{1}{2}\angle ABC$]

From equations (i) and (ii), we get

$$\frac{1}{2}\angle CAB + \frac{1}{2}\angle ABC = \angle BTC + \frac{1}{2}\angle ABC$$

$\Rightarrow \qquad \frac{1}{2}\angle CAB = \angle BTC$

or $\qquad \frac{1}{2}\angle BAC = \angle BTC$

Hence proved.