Exercise 6.4

Question 6:
Prove that a triangle must have at least two acute angles.

Answer 6:
Given:
ΔABC is a triangle

To Prove:
ΔABC must have two acute angles.

Proof:
Let us consider the following cases

Case I
When two angles are 90°
Suppose two angles are $\angle B = 90°$ and $\angle C = 90°$

We know that, the sum of all three angles is 180°
$\therefore\ \angle A + \angle B + \angle C = 180°$
$\therefore\ \angle A + 90° + 90° = 180°$
$\Rightarrow\ \angle A = 180° - 180° = 0$, which is not possible.

Hence, this case is rejected.

Case II
When two angle are obtuse.
Suppose two angles $\angle Band\ \angle C$ are more than 90°

We know that, the sum of all three angles is 180°
$\therefore\ \angle A + \angle B + \angle C = 180°$
$\angle A = 180° - (\angle B + \angle C) = 180° - (\text{Angle greater than } 180°)$
$\angle A =$ negative angle, which is not possible.

Hence, this case is also rejected.

Case III

When one angle in 90° and other is obtuse.
Suppose angles $\angle B = 90°$ and $\angle C$ is obtuse.

We know that, the sum of all three angles is 180°

$\therefore \quad \angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad \angle A = 180° - (90° + \angle C)$

$= 90° - \angle C$

$=$ Negative angle, which is not possible.

Hence, this case is also rejected.

Case IV

When two angles are acute, then sum of two angles is less than180°, so that the third angle is also acute.

Hence, a triangle must have at least two acute angles.