Exercise 6.4

Question 7:

In Fig. 6.17, ∠Q > ∠R, PA is the bisector of ∠QPR and PM ⊥ QR. Prove that ∠APM = $\frac{1}{2}$ (∠Q – ∠R).

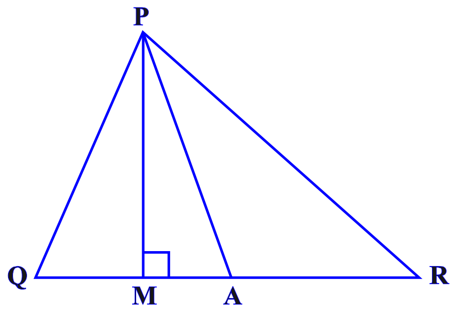

Fig. 6.17

Answer 7:

Given:

ΔPQR , ∠Q > ∠R, PA is the bisector of ∠QPR and PM ⊥ QR.

To prove:

∠APM = $\frac{1}{2}$ (∠Q – ∠R).

Proof:

Since, PA is the bisector of ∠QPR

∴ ∠QPA = ∠APR … (i)

In ΔPQM ∠Q + ∠PMQ + ∠QPM = 180°

[Angle sum property of triangles]

⇒ $\angle Q$ + 90° + ∠QPM = 180°

[$\angle PMQ = 90°$]

⇒ $\angle Q$ = 90° − ∠QPM … (ii)

In ΔPMR, ∠PMR + ∠R + ∠RPM = 180°

[Angle sum property of triangles]

⇒ 90° + $\angle R$ +∠RPM = 180°

[$\angle PMR = 90°$]

⇒ $\angle R$ =180° − 90° − ∠RPM

⇒ $\angle PRM$ =90° − ∠RPM … (iii)

From equations (iii) from and (ii), we get

$$\angle Q - \angle R = (90° - \angle QPM) - (90° - \angle RPM)$$

$\Rightarrow$ $\angle Q - \angle R = \angle RPM - \angle QPM$

$\Rightarrow$ $\angle Q - \angle R = (\angle RPA + \angle APM) - (\angle QPA - \angle APM)$ …(iv)

$\Rightarrow$ $\angle Q - \angle R = \angle QPA + \angle APM - \angle QPA + \angle APM$

[Using equation (i)]

$\Rightarrow$ $\angle Q - \angle R = 2\angle APM$

$\therefore$ $\angle APM = \frac{1}{2}(\angle Q - \angle R)$