

Mathematics

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(Chapter 2)(Inverse Trigonometric Functions)

(Class XII)

(Exemplar Problems)

Objective Type Questions

Choose the correct answers from the given four options in the following (MCQ):

Question 33:

The value of the expression $\tan\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right)$ is

(A) $2 + \sqrt{5}$

(B) $\sqrt{5} - 2$

(C) $\frac{\sqrt{5}+2}{2}$

(D) $5 + \sqrt{2}$

Answer 33:

(B) $\sqrt{5} - 2$

Solution:

To find: $\tan\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right)$

Now, we have $\tan\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right)$

let $\cos^{-1}\frac{2}{\sqrt{5}} = \theta$

$\Rightarrow \tan\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right) = \tan\left(\frac{1}{2}\theta\right)$

$= \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$\left[\because \tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \right]$

$= \sqrt{\frac{1 - \cos\left(\cos^{-1}\frac{2}{\sqrt{5}}\right)}{1 + \cos\left(\cos^{-1}\frac{2}{\sqrt{5}}\right)}}$



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$$= \sqrt{\frac{1 - \frac{2}{\sqrt{5}}}{1 + \frac{2}{\sqrt{5}}}}$$

$$= \sqrt{\frac{\frac{\sqrt{5} - 2}{\sqrt{5}}}{\frac{\sqrt{5} + 2}{\sqrt{5}}}}$$

$$= \sqrt{\frac{\sqrt{5} - 2}{\sqrt{5} + 2}}$$

$$= \sqrt{\frac{\sqrt{5} - 2}{\sqrt{5} + 2} \times \frac{\sqrt{5} - 2}{\sqrt{5} - 2}}$$

$$= \sqrt{\frac{(\sqrt{5} - 2)^2}{5 - 4}} = \sqrt{5} - 2$$

$$\Rightarrow \tan\left(\frac{1}{2} \cos^{-1} \frac{2}{\sqrt{5}}\right) = \sqrt{5} - 2$$

Hence, the option (B) is correct.

