Exercise 2.4

Question 1:

If the polynomials $az^3 + 4z^2 + 3z - 4$ and $z^3 - 4z + a$ leave the same remainder when divided by $z - 3$, find the value of a.

Answer 1:

Given polynomials:

$p(z) = az^3 + 4z^2 + 3z - 4$ and $q(z) = z^3 - 4z + a$

Using remainder theorem,

When $p(z)$ is divided by $z - 3$, the remainder is given by

$$\begin{aligned} p(3) &= a(3)^3 + 4(3)^2 + 3(3) - 4 \\ &= 27a + 36 + 9 - 4 \\ &= 27a + 41 \end{aligned}$$

When $q(z)$ is divided by $z - 3$, the remainder is given by

$$\begin{aligned} q(3) &= (3)^3 - 4(3) + a \\ &= 27 - 12 + a \\ &= 15 + a \end{aligned}$$

According to question:

$p(3) = q(3)$

$\Rightarrow 27a + 41 = 15 + a$

$\Rightarrow 27a - a = 15 - 41$

$\Rightarrow 26a = -26$

$\Rightarrow a = -1$