Exercise 2.4

Question 2:

The polynomial $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$ when divided by $x + 1$ leaves the remainder 19. Find the values of a. Also find the remainder when $p(x)$ is divided by $x + 2$.

Answer 2:

Given polynomials:

$p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$

Using remainder theorem,

When $p(x)$ is divided by $x + 1$, the remainder is given by

$$p(-1) = (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + 3a - 7$$
$$= 1 - 2(-1) + 3 - a(-1) + 3a - 7$$
$$= 1 + 2 + 3 + a + 3a - 7$$
$$= 4a - 1$$

According to question:

$p(-1) = 19$

$\Longrightarrow 4a - 1 = 19$

$\Longrightarrow 4a = 20$

$\Longrightarrow a = 5$

Hence, $p(x) = x^4 - 2x^3 + 3x^2 - 5x + 15 - 7 = x^4 - 2x^3 + 3x^2 - 5x + 8$

When $p(x)$ is divided by $x + 2$, the remainder is given by

$$p(-2) = (-2)^4 - 2(-2)^3 + 3(-2)^2 - 5(-2) + 8$$
$$= 16 - 2(-8) + 3(4) + 10 + 8$$
$$= 16 + 16 + 12 + 18 = 62$$