Exercise 2.4

Question 4:

Without actual division, prove that $2x^4 - 5x^3 + 2x^2 - x + 2$ is divisible by $x^2 - 3x + 2$. [Hint: Factorise $x^2 - 3x + 2$]

Answer 4:

Given polynomials:

$f(x) = 2x^4 - 5x^3 + 2x^2 - x + 2$ and $g(x) = x^2 - 3x + 2$

Here, $g(x) = x^2 - 3x + 2$

$= x^2 - x - 2x + 2$

$= x(x - 1) - 2(x - 1)$

$= (x - 2)(x - 1)$

Now, we have to show that $x - 2$ and $x - 1$ are the factors of $f(x)$.

Using factor theorem,

When $f(x)$ is divided by $x - 2$, the remainder is given by

$f(2) = 2(2)^4 - 5(2)^3 + 2(2)^2 - (2) + 2$

$= 32 - 40 + 8 - 2 + 2$

$= 42 - 42$

$= 0$

$\Rightarrow$ $x - 2$ is a factor of $f(x)$ ... (i)

Similarly, when $f(x)$ is divided by $x - 1$, the remainder is given by

$f(1) = 2(1)^4 - 5(1)^3 + 2(1)^2 - (1) + 2$

$= 2 - 5 + 2 - 1 + 2$

$= 6 - 6$

$= 0$

$\Rightarrow$ $x - 1$ is a factor of $f(x)$ ... (ii)

From equations (i) and (ii), we get

$x - 2$ and $x - 1$ are the factors of $f(x)$

$\Rightarrow x^2 - 3x + 2$ is a factor of $f(x)$.

Hence, $f(x) = 2x^4 - 5x^3 + 2x^2 - x + 2$ is divisible by $x^2 - 3x + 2$.