Exercise 2.3

Question 24:

Factorise:

(i) $2x^3 - 3x^2 - 17x + 30$ **(ii)** $x^3 - 6x^2 + 11x - 6$

(iii) $x^3 + x^2 - 4x - 4$ **(iv)** $3x^3 - x^2 - 3x + 1$

Answer 24:

(i). Given that: $p(x) = 2x^3 - 3x^2 - 17x + 30$

$\Rightarrow p(1) = 2(1)^3 - 3(1)^2 - 17(1) + 30$

$= 2 - 3 - 17 + 30$

$= 32 - 20 = 12 \neq 0$

$p(2) = 2(2)^3 - 3(2)^2 - 17(2) + 30$

$= 16 - 12 - 34 + 30$

$= 46 - 46 = 0$

$\Rightarrow (x - 2)$ is a factor of $p(x)$.

$$
\begin{array}{r|l}
 & 2x^2 + x - 15 \\
\hline
x - 2 & 2x^3 - 3x^2 - 17x + 30 \\
 & 2x^3 - 4x^2 \\
 & - \quad + \\
\hline
 & \quad x^2 - 17x + 30 \\
 & \quad x^2 - \quad 2x \\
 & \quad - \quad + \\
\hline
 & \quad\quad - 15x + 30 \\
 & \quad\quad - 15x + 30 \\
 & \quad\quad + \quad - \\
\hline
 & \quad\quad\quad 0
\end{array}
$$

Now, $p(x) = (x - 2)(2x^2 + x - 15)$

$= (x - 2)(2x^2 + 6x - 5x - 15)$

$= (x - 2)[2x(x + 3) - 5(x + 3)]$

$= (x - 2)[(x + 3)(2x - 5)]$

$= (x - 2)(x + 3)(2x - 5)$

(ii). Given that: $p(x) = x^3 - 6x^2 + 11x - 6$

$\Rightarrow p(1) = (1)^3 - 6(1)^2 + 11(1) - 6$

$= 1 - 6 + 11 - 6$

$= 12 - 12$

$= 0$

$\Rightarrow (x - 1)$ is a factor of $p(x)$.

$$
\begin{array}{r|l}
 & x^2 - 5x + 6 \\
\hline
x - 1 & x^3 - 6x^2 + 11x - 6 \\
 & x^3 - x^2 \\
 & - \quad + \\
\hline
 & -5x^2 + 11x - 6 \\
 & -5x^2 + 5x \\
 & + \quad - \\
\hline
 & +6x - 6 \\
 & +6x - 6 \\
 & - \quad + \\
\hline
 & 0
\end{array}
$$

Now, $p(x) = (x - 1)(x^2 - 5x + 6)$

$= (x - 1)(x^2 - 3x - 2x + 6)$

$= (x - 1)[x(x - 3) - 2(x - 3)]$

$= (x - 1)[(x - 3)(x - 2)]$

$= (x - 1)(x - 2)(x - 3)$

(iii). Given that: $p(x) = x^3 + x^2 - 4x - 4$

$\Rightarrow p(1) = (1)^3 + (1)^2 - 4(1) - 4$

$= 1 + 1 - 4 - 4$

$= 2 - 8$

$= -6 \neq 0$

$p(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4$
$= -1 + 1 + 4 - 4$
$= 5 - 5 = 0$
$\Rightarrow (x + 1)$ is a factor of $p(x)$.

$$
\begin{array}{r|l}
 & x^2 - 4 \\
\hline
x + 1 & x^3 + x^2 - 4x - 4 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & -4x - 4 \\
 & -4x - 4 \\
 & + \quad + \\
\hline
 & 0
\end{array}
$$

Now, $p(x) = (x + 1)(x^2 - 4)$
$= (x + 1)(x^2 - 2^2)$
$= (x + 1)[(x - 2)(x + 2)]$
$= (x + 1)(x - 2)(x + 2)$

(iv). Given that: $p(x) = 3x^3 - x^2 - 3x + 1$
$\Rightarrow p(1) = 3(1)^3 - (1)^2 - 3(1) + 1$
$= 3 - 1 - 3 + 1$
$= 4 - 4 = 0$
$\Rightarrow (x - 1)$ is a factor of $p(x)$.

$$
\begin{array}{r|l}
 & 3x^2 + 2x - 1 \\
\hline
x - 1 & 3x^3 - x^2 - 3x + 1 \\
 & 3x^3 - 3x^2 \\
 & - \quad + \\
\hline
 & 2x^2 - 3x + 1 \\
 & 2x^2 - 2x \\
 & - \quad + \\
\hline
 & -x + 1 \\
 & -x + 1 \\
 & + \quad - \\
\hline
 & 0
\end{array}
$$

Now, $p(x) = (x-1)(3x^2+2x-1)$
$= (x-1)(3x^2+3x-x-1)$
$= (x-1)[3x(x+1)-1(x+1)]$
$= (x-1)[(x+1)(3x-1)]$
$= (x-1)(x+1)(3x-1)$