Question 1:
Find the principal value of \( \sin^{-1}\left(-\frac{1}{2}\right) \)

Answer 1:
Let \( \sin^{-1}\left(-\frac{1}{2}\right) = y \), then \( \sin y = -\frac{1}{2} = -\sin \left(\frac{\pi}{6}\right) = \sin \left(-\frac{\pi}{6}\right) \)

We know that the range of the principal value branch of \( \sin^{-1} \) is \( \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \) and \( \sin \left(-\frac{\pi}{6}\right) = -\frac{1}{2} \)

Therefore, the principal value of \( \sin^{-1}\left(-\frac{1}{2}\right) \) is \( -\frac{\pi}{6} \).

Question 2:
Find the principal value of \( \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \)

Answer 2:
Let \( \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = y \), then \( \cos y = \frac{\sqrt{3}}{2} = \cos \left(\frac{\pi}{6}\right) \)

We know that the range of the principal value branch of \( \cos^{-1} \) is \( [0, \pi] \) and \( \cos \left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \)

Therefore, the principal value of \( \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \) is \( \frac{\pi}{6} \).

Question 3:
Find the principal value of \( \csc^{-1}(2) \)

Answer 3:
Let \( \csc^{-1}(2) = y \), then \( \csc y = 2 = \csc \left(\frac{\pi}{6}\right) \)

We know that the range of the principal value branch of \( \csc^{-1} \) is \( \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \)-\( \{0\} \) and \( \csc \left(\frac{\pi}{6}\right) = 2 \).

Therefore, the principal value of \( \csc^{-1}(2) \) is \( \frac{\pi}{6} \).

Question 4:
Find the principal value of \( \tan^{-1}(-\sqrt{3}) \).

Answer 4:
Let \( \tan^{-1}(-\sqrt{3}) = y \), then \( \tan y = -\sqrt{3} = -\tan \left(\frac{\pi}{3}\right) = \tan \left(-\frac{\pi}{3}\right) \)

We know that the range of the principal value branch of \( \tan^{-1} \) is \( \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \) and \( \tan \left(-\frac{\pi}{3}\right) = -\sqrt{3} \)

Therefore, the principal value of \( \tan^{-1}(-\sqrt{3}) \) is \( -\frac{\pi}{3} \).
Question 5:
Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$.

Answer 5:
Let $\cos^{-1}\left(-\frac{1}{2}\right) = y$, then $\cos y = -\frac{1}{2} = -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$
We know that the range of the principal value branch of $\cos^{-1}$ is $[0, \pi]$ and $
\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$
Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

Question 6:
Find the principal value of $\tan^{-1}(-1)$.

Answer 6:
Let $\tan^{-1}(-1) = y$. Then, $\tan y = -1 = -\tan\left(\frac{\pi}{4}\right) = \tan\left(-\frac{\pi}{4}\right)$
We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{4}\right) = -1$
Therefore, the principal value of $\tan^{-1}(-1)$ is $-\frac{\pi}{4}$.

Question 7:
Find the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$.

Answer 7:
Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$, then $\sec y = \frac{2}{\sqrt{3}} = \sec\left(\frac{\pi}{6}\right)$
We know that the range of the principal value branch of $\sec^{-1}$ is $[0, \pi) - \left\{\frac{\pi}{2}\right\}$ and $\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$
Therefore, the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$.

Question 8:
Find the principal value of $\cot^{-1}\sqrt{3}$.

Answer 8:
Let $\cot^{-1}\sqrt{3} = y$, then $\cot y = \sqrt{3} = \cot\left(\frac{\pi}{6}\right)$.
We know that the range of the principal value branch of $\cot^{-1}$ is $(0, \pi)$ and $\cot\left(\frac{\pi}{6}\right) = \sqrt{3}$.
Therefore, the principal value of $\cot^{-1}\sqrt{3}$ is $\frac{\pi}{6}$.
Question 9:
Find the principal value of \( \cos^{-1}\left( -\frac{1}{\sqrt{2}} \right) \).

**Answer 9:**
Let \( \cos^{-1}\left( -\frac{1}{\sqrt{2}} \right) = y \), then \( \cos y = -\frac{1}{\sqrt{2}} = -\cos \left( \frac{\pi}{4} \right) = \cos \left( \pi - \frac{\pi}{4} \right) = \cos \left( \frac{3\pi}{4} \right) \).
We know that the range of the principal value branch of \( \cos^{-1} \) is \([0, \pi]\) and \( \cos \left( \frac{3\pi}{4} \right) = -\frac{1}{\sqrt{2}} \).
Therefore, the principal value of \( \cos^{-1}\left( -\frac{1}{\sqrt{2}} \right) \) is \( \frac{3\pi}{4} \).

Question 10:
Find the principal value of \( \csc^{-1}\left( -\sqrt{2} \right) \).

**Answer 10:**
Let \( \csc^{-1}\left( -\sqrt{2} \right) = y \), then \( \csc y = -\sqrt{2} = -\csc \left( \frac{\pi}{4} \right) = \csc \left( -\frac{\pi}{4} \right) \).
We know that the range of the principal value branch of \( \csc^{-1} \) is \( \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] - \{0\} \) and \( \csc \left( -\frac{\pi}{4} \right) = -\sqrt{2} \).
Therefore, the principal value of \( \csc^{-1}\left( -\sqrt{2} \right) \) is \( -\frac{\pi}{4} \).

Question 11:
Find the value of \( \tan^{-1}(1) + \cos^{-1}\left( -\frac{1}{2} \right) + \sin^{-1}\left( -\frac{1}{2} \right) \).

**Answer 11:**
Let \( \tan^{-1}(1) = x \), then \( \tan x = 1 = \tan \frac{\pi}{4} \).
We know that the range of the principal value branch of \( \tan^{-1} \) is \( \left( -\frac{\pi}{2}, \frac{\pi}{2} \right) \).
\[ \therefore \tan^{-1}(1) = \frac{\pi}{4} \]
Let \( \cos^{-1}\left( -\frac{1}{2} \right) = y \), then
\[ \cos y = -\frac{1}{2} = -\cos \frac{\pi}{3} = \cos \left( \pi - \frac{\pi}{3} \right) = \cos \left( \frac{2\pi}{3} \right) \]
We know that the range of the principal value branch of \( \cos^{-1} \) is \([0, \pi]\).
\[ \therefore \cos^{-1}\left( -\frac{1}{2} \right) = \frac{2\pi}{3} \]
Let \( \sin^{-1}\left( -\frac{1}{2} \right) = z \), then
\[ \sin z = -\frac{1}{2} = -\sin \frac{\pi}{6} = \sin \left( -\frac{\pi}{6} \right) \]
We know that the range of the principal value branch of $\sin^{-1}$ is $\left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$.

$\therefore \sin^{-1} \left( -\frac{1}{2} \right) = -\frac{\pi}{6}$

Now,

$\tan^{-1}(1) + \cos^{-1} \left( -\frac{1}{2} \right) + \sin^{-1} \left( -\frac{1}{2} \right)$

$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$

$= \frac{3\pi + 8\pi - 2\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$

**Question 12:**
Find the value of $\cos^{-1} \left( \frac{1}{2} \right) + 2\sin^{-1} \left( \frac{1}{2} \right)$

**Answer 12:**
Let $\cos^{-1} \left( \frac{1}{2} \right) = x$, then

$\cos x = \frac{1}{2} = \cos \frac{\pi}{3}$

We know that the range of the principal value branch of $\cos^{-1}$ is $[0, \pi]$.

$\therefore \cos^{-1} \left( \frac{1}{2} \right) = \frac{\pi}{3}$

Let $\sin^{-1} \left( -\frac{1}{2} \right) = y$, then

$\sin y = \frac{1}{2} = \sin \frac{\pi}{6}$

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$.

$\therefore \sin^{-1} \left( \frac{1}{2} \right) = \frac{\pi}{6}$

Now,

$\cos^{-1} \left( \frac{1}{2} \right) + 2\sin^{-1} \left( \frac{1}{2} \right) = \frac{\pi}{3} + 2 \times \frac{\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$.

**Question 13:**
If $\sin^{-1} x = y$, then

(A) $0 \leq y \leq \pi$

(B) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

(C) $0 < y < \pi$

(D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

**Answer 13:**
It is given that $\sin^{-1} x = y$.

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$.

Therefore, $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.

Hence, the option (B) is correct.
Question 14:
\( \tan^{-1}\sqrt{3} - \sec^{-1}(-2) \) is equal to
(A) \( \pi \)  
(B) \( -\frac{\pi}{3} \)  
(C) \( \frac{\pi}{3} \)  
(D) \( \frac{2\pi}{3} \)

Answer 14:
Let \( \tan^{-1}\sqrt{3} = x \), then
\[
\tan x = \sqrt{3} = \tan \frac{\pi}{3}
\]
We know that the range of the principal value branch of \( \tan^{-1} \) is \( \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \).
∴ \( \tan^{-1}\sqrt{3} = \frac{\pi}{3} \)

Let \( \sec^{-1}(-2) = y \), then
\[
\sec y = -2 = -\sec \frac{\pi}{3} = \sec \left(\pi - \frac{\pi}{3}\right) = \sec \left(\frac{2\pi}{3}\right)
\]
We know that the range of the principal value branch of \( \sec^{-1} \) is \([0, \pi] - \left\{ \frac{\pi}{2} \right\} \).
∴ \( \sec^{-1}(-2) = \frac{2\pi}{3} \)

Now,
\[
\tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}
\]
Hence, the option (B) is correct.