Exercise 4.4

Write Minors and Cofactors of the elements of following determinants:

Question 1:

(i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$ **(ii)** $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

Answer 1:

(i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$

The minor of element a_{ij} is M_{ij} and the cofactor is $A_{ij} = (-1)^{i+j}M_{ij}$, therefore,

The minor of element a_{11} is M_{11}= 3 and the cofactor is $A_{11} = (-1)^{1+1}M_{11} = 3$

The minor of element a_{12} is M_{12}= 0 and the cofactor is $A_{12} = (-1)^{1+2}M_{12} = 0$

The minor of element a_{21} is M_{21}= -4 and the cofactor is $A_{21} = (-1)^{2+1}M_{21} = 4$

The minor of element a_{22} is M_{22}= 2 and the cofactor is $A_{22} = (-1)^{2+2}M_{22} = 2$

(ii) $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

The minor of element a_{11} is M_{11}= d and the cofactor is $A_{11} = (-1)^{1+1}M_{11} = d$

The minor of element a_{12} is M_{12}= b and the cofactor is $A_{12} = (-1)^{1+2}M_{12} = -b$

The minor of element a_{21} is M_{21}= c and the cofactor is $A_{21} = (-1)^{2+1}M_{21} = -c$

The minor of element a_{22} is M_{22}= a and the cofactor is $A_{22} = (-1)^{2+2}M_{22} = a$

Question 2:

(i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ **(ii)** $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

Answer 2:

(i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$,

Here,

$M_{11} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$, $M_{12} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0$, $M_{13} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0$

$M_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0$, $M_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$, $M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$

$M_{31} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0 - 0 = 0$, $M_{32} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$, $M_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$

and $A_{ij} = (-1)^{i+j}M_{ij}$, therefore

$A_{11} = (-1)^{1+1}M_{11} = 1$ $A_{12} = (-1)^{1+2}M_{12} = 0$ $A_{13} = (-1)^{1+3}M_{13} = 0$

$A_{21} = (-1)^{2+1}M_{21} = 0$ $A_{22} = (-1)^{2+2}M_{22} = 1$ $A_{23} = (-1)^{2+3}M_{233} = 0$

$A_{31} = (-1)^{3+1}M_{31} = 0$ $A_{32} = (-1)^{3+2}M_{32} = 0$ $A_{33} = (-1)^{3+3}M_{33} = 1$

(ii) $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$,

Here,

$M_{11} = \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 10 + 1 = 11,\ \ M_{12} = \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 6 - 0 = 6,\ M_{13} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$

$M_{21} = \begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = 0 - 4 = -4,\ \ \ M_{22} = \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2,\ \ M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$

$M_{31} = \begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = 0 - 20 = -20, M_{32} = \begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -1 - 12 = -13\ M_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5 - 0 = 5$

and $A_{ij} = (-1)^{i+j} M_{ij}$, therefore

$A_{11} = (-1)^{1+1} M_{11} = 11 \qquad A_{12} = (-1)^{1+2} M_{12} = -6 \qquad A_{13} = (-1)^{1+3} M_{13} = 3$

$A_{21} = (-1)^{2+1} M_{21} = 4 \qquad A_{22} = (-1)^{2+2} M_{22} = 2 \qquad A_{23} = (-1)^{2+3} M_{233} = -1$

$A_{31} = (-1)^{3+1} M_{31} = -20 \qquad A_{32} = (-1)^{3+2} M_{32} = 13 \qquad A_{33} = (-1)^{3+3} M_{33} = 5$

Question 3:

Using Cofactors of elements of second row, evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.

Answer 3:

$\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix} = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$

Here, $a_{21} = 2,\ \ a_{22} = 0,\ \ a_{23} = 1$ and

$A_{21} = (-1)^{2+1} \begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = -(9 - 16) = 7$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} = 15 - 8 = 7$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = -(10 - 3) = -7$

Therefore, $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 2(7) + 0(7) + 1(-7) = 7$

Question 4:

Using Cofactors of elements of third column, evaluate $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$.

Answer 4:

$\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix} = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$

Here, $a_{13} = yz,\ \ a_{23} = zx,\ \ a_{33} = xy$ and

$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix} = z - y$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix} = -(z - x) = x - z$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix} = y - x$

Therefore, $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix} = yz(z-y) + zx(x-z) + xy(y-x)$

$= yz^2 - y^2z + zx^2 - xz^2 + xy^2 - x^2y$
$= zx^2 - x^2y - xz^2 + xy^2 + yz^2 - y^2z$
$= x^2(z-y) - x(z^2-y^2) + yz(z-y)$
$= (z-y)[x^2 - x(z+y) + yz]$
$= (z-y)[x^2 - xz - xy + yz]$
$= (z-y)[x(x-z) - y(x-z)]$
$= (x-z)(z-y)(x-y)$
$= (x-y)(y-z)(z-x)$

Question 5:

If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ A_{ij} is Cofactors of a_{ij}, then value of Δ is given by

(A) $a_{11}A_{11} + a_{12}A_{32} + a_{13}A_{33}$
(B) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$
(C) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$
(D) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Answer 5:

The value of $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ is given by: $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Hence, the option (D) is correct.