Mathematics

(Chapter - 5) (Continuity and Differentiability)

(Class 12)

Exercise 5.1

Question 1:
Prove that the function \( f(x) = 5x - 3 \) is continuous at \( x = 0 \), at \( x = -3 \) and at \( x = 5 \).

Answer 1:
Given function \( f(x) = 5x - 3 \)
At \( x = 0 \), \( f(0) = 5(0) - 3 = -3 \)
LHL = \( \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (5x - 3) = -3 \)
RHL = \( \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (5x - 3) = -3 \)
Here, at \( x = 0 \), LHL = RHL = \( f(0) = -3 \)
Hence, the function \( f \) is continuous at \( x = 0 \).

At \( x = -3 \), \( f(-3) = 5(-3) - 3 = -18 \)
LHL = \( \lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} (5x - 3) = -18 \)
RHL = \( \lim_{x \to -3^+} f(x) = \lim_{x \to -3^+} (5x - 3) = -18 \)
Here, at \( x = -3 \), LHL = RHL = \( f(-3) = -18 \)
Hence, the function \( f \) is continuous at \( x = -3 \).

At \( x = 5 \), \( f(5) = 5(5) - 3 = 22 \)
LHL = \( \lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} (5x - 3) = 22 \)
RHL = \( \lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} (5x - 3) = 22 \)
Here, at \( x = 5 \), LHL = RHL = \( f(5) = 22 \)
Hence, the function \( f \) is continuous at \( x = 5 \).

Question 2:
Examine the continuity of the function \( f(x) = 2x^2 - 1 \) at \( x = 3 \).

Answer 2:
Given function \( f(x) = 2x^2 - 1 \). At \( x = 3 \), \( f(3) = 2(3)^2 - 1 = 17 \)
LHL = \( \lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (2x^2 - 1) = 17 \)
RHL = \( \lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (2x^2 - 1) = 17 \)
Here, at \( x = 3 \), LHL = RHL = \( f(3) = 17 \)
Hence, the function \( f \) is continuous at \( x = 3 \).

Question 3:
Examine the following functions for continuity:

(a) \( f(x) = x - 5 \)
(b) \( f(x) = \frac{1}{x-5}, x \neq 5 \)
(c) \( f(x) = \frac{x^2-25}{x+5}, x \neq -5 \)
(d) \( f(x) = |x - 5| \)

Answer 3:
(a) Given function \( f(x) = x - 5 \)
Let, \( k \) be any real number. At \( x = k \), \( f(k) = k - 5 \)
LHL = \( \lim_{x \to k^-} f(x) = \lim_{x \to k^-} (x - 5) = k - 5 \)
RHL = \( \lim_{x \to k^+} f(x) = \lim_{x \to k^+} (x - 5) = k - 5 \)
At, \( x = k \), LHL = RHL = \( f(k) = k - 5 \)
Hence, the function \( f \) is continuous for all real numbers.
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(b) Given function \( f(x) = \frac{1}{x-5}, x \neq 5 \)

Let, \( k \ (k \neq 5) \) be any real number. At \( x = k, f(k) = \frac{1}{k-5} \)

\[
\text{LHL} = \lim_{x \to k^-} f(x) = \lim_{x \to k^-} \left( \frac{1}{x-5} \right) = \frac{1}{k-5}
\]

\[
\text{RHL} = \lim_{x \to k^+} f(x) = \lim_{x \to k^+} \left( \frac{1}{x-5} \right) = \frac{1}{k-5}
\]

At, \( x = k, \text{LHL} = \text{RHL} = f(k) = \frac{1}{k-5} \)

Hence, the function \( f \) is continuous for all real numbers (except 5).

(c) Given function \( f(x) = \frac{x^2-25}{x+5}, x \neq -5 \)

Let, \( k \ (k \neq -5) \) be any real number.

At \( x = k, f(k) = \frac{k^2-25}{k+5} = \frac{(k+5)(k-5)}{(k+5)} = (k + 5) \)

\[
\text{LHL} = \lim_{x \to k^-} f(x) = \lim_{x \to k^-} \left( \frac{x^2-25}{x+5} \right) = \lim_{x \to k^-} \left( \frac{(k+5)(k-5)}{(k+5)} \right) = k + 5
\]

\[
\text{RHL} = \lim_{x \to k^+} f(x) = \lim_{x \to k^+} \left( \frac{x^2-25}{x+5} \right) = \lim_{x \to k^+} \left( \frac{(k+5)(k-5)}{(k+5)} \right) = k + 5
\]

At, \( x = k, \text{LHL} = \text{RHL} = f(k) = k + 5 \)

Hence, the function \( f \) is continuous for all real numbers (except -5).

(d) Given function \( f(x) = |x - 5| \)

Let, \( k \) be any real number. According to question, \( k < 5 \) or \( k = 5 \) or \( k > 5 \).

First case: If, \( k < 5 \),

\( f(k) = 5 - k \) and \( \lim_{x \to k^-} f(x) = \lim_{x \to k^-} (5 - x) = 5 - k \)

Hence, the function \( f \) is continuous for all real numbers less than 5.

Second case: If, \( k = 5 \),

\( f(k) = 5 - 5 \) and \( \lim_{x \to k^-} f(x) = \lim_{x \to k^-} (x - 5) = 5 - k \)

Hence, the function \( f \) is continuous at \( x = 5 \).

Third case: If, \( k > 5 \),

\( f(k) = k - 5 \) and \( \lim_{x \to k^-} f(x) = \lim_{x \to k^-} (x - 5) = 5 - k \)

Hence, the function \( f \) is continuous for all real numbers greater than 5.

Hence, the function \( f \) is continuous for all real numbers.

Question 4:
Prove that the function \( f(x) = x^n \), is continuous at \( x = n \), where \( n \) is a positive integer.

Answer 4:
Given function \( f(x) = x^n \).

At \( x = n, f(n) = n^n \)

\[
\lim_{x \to n} f(x) = \lim_{x \to n} (x^n) = n^n
\]

Here, at \( x = n, \lim_{x \to n} f(x) = f(n) = n^n \)

Hence, the function \( f \) is continuous at \( x = n \), where \( n \) is a positive integer.

Question 5:
Is the function \( f(x) \) defined by \( f(x) = \begin{cases} x, & x \leq 1 \\ 5, & x > 1 \end{cases} \) continuous at \( x = 0 \)? At \( x = 1 \)? At \( x = 2 \)?
**Answer 5:**

Given function \( f(x) = \begin{cases} x, & x \leq 1 \\ 5, & x > 1 \end{cases} \)

At \( x = 0, f(0) = 0 \)

\[ \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x = 0 \]

Here, \( x = 0, \lim_{x \to 0^-} f(x) = f(0) = 0 \)

Hence, the function \( f \) is discontinuous at \( x = 0 \).

At \( x = 1, f(1) = 1 \)

LHL = \( \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x = 1 \)

RHL = \( \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 5 = 5 \)

Here, at \( x = 1 \), LHL \( \neq \) RHL. Hence, the function \( f \) is discontinuous at \( x = 1 \).

At \( x = 2, f(2) = 5 \)

\[ \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} 5 = 5 \]

Here, at \( x = 2, \lim_{x \to 2^-} f(x) = f(2) = 5 \)

Hence, the function \( f \) is continuous at \( x = 2 \).

**Question 6:**

Find all points of discontinuity of \( f \), where \( f \) is defined by

\[ f(x) = \begin{cases} 2x + 3, & \text{if } x \leq 2 \\ 2x - 3, & \text{if } x > 2 \end{cases} \]

**Answer 6:**

Let, \( k \) be any real number. According to question, \( k < 2 \) or \( k = 2 \) or \( k > 2 \)

**First case:** यदि, \( k < 2 \),

\[ f(k) = 2k + 3 \text{ and } \lim_{x \to k^-} f(x) = \lim_{x \to k^-} (2x + 3) = 2k + 3 \]

Here, \( \lim_{x \to k^-} f(x) = f(k) \)

Hence, the function \( f \) is continuous for all real numbers smaller than 2.

**Second case:** If \( k = 2, f(2) = 2k + 3 \)

LHL = \( \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2x + 3) = 7 \)

RHL = \( \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (2x - 3) = 1 \)

Here, at \( x = 2, \text{ LHL} \neq \text{ RHL} \). Hence, the function \( f \) is discontinuous at \( x = 2 \).

**Third case:** If \( k > 2 \),

\[ f(k) = 2k - 3 \text{ and } \lim_{x \to k^-} f(x) = \lim_{x \to k^-} (2x - 3) = 2k - 3 \]

Here, \( \lim_{x \to k^-} f(x) = f(k) \)

Therefore, the function \( f \) is continuous for all real numbers greater than 2.

Hence, the function \( f \) is discontinuous only at \( x = 2 \).

**Question 7:**

\[ f(x) = \begin{cases} |x| + 3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x + 2, & \text{if } x \geq 3 \end{cases} \]

**Answer 7:**

Let, \( k \) be any real number. According to question,

\( k < -3 \) or \( k = -3 \) or \(-3 < k < 3 \) or \( k = 3 \) or \( k > 3 \)
First case: If \( k < -3 \),
\[
f(k) = -k + 3 \text{ and } \lim_{x \to k} f(x) = \lim_{x \to k} (-x + 3) = -k + 3. \text{ Here, } \lim_{x \to k} f(x) = f(k)
\]
Hence, the function \( f \) is continuous for all real numbers less than \( -3 \).

Second case: If \( k = -3 \), \( f(-3) = -(3) + 3 = 6 \)
\[
\lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} (-x + 3) = -(3) + 3 = 6
\]
\[
\lim_{x \to -3^+} f(x) = \lim_{x \to -3^+} (-2x) = -2(-3) = 6. \text{ Here, } \lim_{x \to -3} f(x) = f(k)
\]
Hence, the function \( f \) is continuous at \( x = -3 \).

Third case: If \( -3 < k < 3 \),
\[
f(k) = -2k \text{ and } \lim_{x \to k} f(x) = \lim_{x \to k} (-2x) = -2k. \text{ Here, } \lim_{x \to k} f(x) = f(k)
\]
Hence, the function \( f \) is continuous at \(-3 < x < 3\).

Fourth case: If \( k = 3 \),
\[
LHL = \lim_{x \to k^-} f(x) = \lim_{x \to k^-} (-2x) = -2k
\]
\[
RHL = \lim_{x \to k^+} f(x) = \lim_{x \to k^+} (6x + 2) = 6k + 2,
\]
Here, at \( x = 3 \), LHL \( \neq \) RHL. Hence, the function \( f \) is discontinuous at \( x = 3 \).

Fifth case: If \( k > 3 \),
\[
f(k) = 6k + 2 \text{ and } \lim_{x \to k} f(x) = \lim_{x \to k} (6x + 2) = 6k + 2. \text{ Here, } \lim_{x \to k} f(x) = f(k)
\]
Hence, the function \( f \) is continuous for all numbers greater than \( 3 \).
Hence, the function \( f \) is discontinuous only at \( x = 3 \).

**Question 8:**

\[
f(x) = \begin{cases} 
  \frac{|x|}{x}, & \text{if } x \neq 0 \\
  0, & \text{if } x = 0 
\end{cases}
\]

**Answer 8:**

After redefining the function \( f \), we get
\[
f(x) = \begin{cases} 
  -\frac{x}{x}, & \text{if } x < 0 \\
  \frac{x}{x}, & \text{if } x = 0 \\
  \frac{x}{x}, & \text{if } x > 0 
\end{cases}
\]
Let, \( k \) be any real number. According to question, \( k < 0 \) or \( k = 0 \) or \( k > 0 \).

First case: If \( k < 0 \),
\[
f(k) = -\frac{k}{k} = -1 \text{ and } \lim_{x \to k} f(x) = \lim_{x \to k} \left(-\frac{x}{x}\right) = -1. \text{ Here, } \lim_{x \to k} f(x) = f(k)
\]
Hence, the function \( f \) is continuous for all real numbers smaller than \( 0 \).

Second case: If \( k = 0 \), \( f(0) = 0 \)
\[
LHL = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \left(-\frac{x}{x}\right) = -1 \quad \text{and} \quad RHL = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \left(\frac{x}{x}\right) = 1,
\]
Here, at \( x = 0 \), LHL \( \neq \) RHL. Hence, the function \( f \) is discontinuous at \( x = 0 \).

Third case: If \( k > 0 \),
\[
f(k) = \frac{k}{k} = 1 \text{ and } \lim_{x \to k} f(x) = \lim_{x \to k} \left(\frac{x}{x}\right) = 1. \text{ Here, } \lim_{x \to k} f(x) = f(k)
\]
Hence, the function \( f \) is continuous for all real numbers greater than \( 0 \).
Therefore, the function \( f \) is discontinuous only at \( x = 0 \).

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Question 9:

\[ f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x < 0 \\ -1, & \text{if } x \geq 0 \end{cases} \]

Answer 9:
Redefining the function, we get

\[ f(x) = \begin{cases} \frac{x}{|x|} = \frac{x}{-x} = -1, & \text{if } x < 0 \\ -1, & \text{if } x \geq 0 \end{cases} \]

Here, \( \lim_{x \to k} f(x) = -1 \), where \( k \) is a real number.
Hence, the function \( f \) is continuous for all real numbers.

Question 10:

\[ f(x) = \begin{cases} x + 1, & \text{if } x \geq 1 \\ x^2 + 1, & \text{if } x < 1 \end{cases} \]

Answer 10:
Let, \( k \) be any real number. According to question, \( k < 1 \) or \( k = 1 \) or \( k > 1 \)

First case: If, \( k < 1 \),
\[ f(k) = k^2 + 1 \text{ and } \lim_{x \to k} f(x) = \lim_{x \to k} (x^2 + 1) = k^2 + 1. \]
Here, \( \lim_{x \to k} f(x) = f(k) \)
Hence, the function \( f \) is continuous for all real numbers smaller than 1.

Second case: If, \( k = 1 \), \( f(1) = 1 + 1 = 2 \)
LHL = \( \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x^2 + 1) = 1 + 1 = 2 \)
RHL = \( \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x + 1) = 1 + 1 = 2 \),
Here, at \( x = 1 \), LHL = RHL = \( f(1) \). Hence, the function \( f \) is continuous at \( x = 1 \).

Third case: If, \( k > 1 \),
\[ f(k) = k + 1 \text{ and } \lim_{x \to k} f(x) = \lim_{x \to k} (x + 1) = k + 1. \]
Here, \( \lim_{x \to k} f(x) = f(k) \)
Hence, the function \( f \) is continuous for all real numbers greater than 1.
Therefore, the function \( f \) is continuous for all real numbers.

Question 11:

\[ f(x) = \begin{cases} x^3 - 3, & \text{if } x \leq 2 \\ x^2 + 1, & \text{if } x > 2 \end{cases} \]

Answer 11:
Let, \( k \) be any real number. According to question, \( k < 2 \) or \( k = 2 \) or \( k > 2 \)

First case: If, \( k < 2 \),
\[ f(k) = k^3 - 3 \text{ and } \lim_{x \to k} f(x) = \lim_{x \to k} (x^3 - 3) = k^3 - 3. \]
Here, \( \lim_{x \to k} f(x) = f(k) \)
Hence, the function \( f \) is continuous for all real numbers less than 2.

Second case: If, \( k = 2 \), \( f(2) = 2^3 - 3 = 5 \)
LHL = \( \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x^3 - 3) = 2^3 - 3 = 5 \)
RHL = \( \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x^2 + 1) = 2^2 + 1 = 5 \),
Here, at \( x = 2 \), LHL = RHL = \( f(2) \)
Hence, the function \( f \) is continuous at \( x = 2 \).
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Third case: If, \( k > 2 \),
\[ f(k) = k^2 + 1 \text{ and } \lim_{x \to k} f(x) = \lim_{x \to k} (x^2 + 1) = k^2 + 1. \]
Here, \( \lim_{x \to k} f(x) = f(k) \)
Hence, the function \( f \) is continuous for real numbers greater than 2.
Hence, the function \( f \) is continuous for all real numbers.

**Question 12:**

\[
\begin{cases}
  x^{10} - 1, & \text{if } x \leq 1 \\
  x^2, & \text{if } x > 1 
\end{cases}
\]

**Answer 12:**
Let, \( k \) be any real number. According to question, \( k < 1 \) or \( k = 1 \) or \( k > 1 \)

First case: If, \( k < 1 \),
\[ f(k) = k^{10} - 1 \text{ and } \lim_{x \to k} f(x) = \lim_{x \to k} (x^{10} - 1) = k^{10} - 1. \]
Here, \( \lim_{x \to k} f(x) = f(k) \)
Hence, the function \( f \) is continuous for all real numbers less than 1.

Second case: If, \( k = 1 \), \( f(1) = 1^{10} - 1 = 0 \)
LHL = \( \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x^{10} - 1) = 0 \)
RHL = \( \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x^2) = 1, \)
Here, at \( x = 1 \), LHL \( \neq \) RHL. Hence, the function \( f \) is discontinuous at \( x = 1 \).

Third case: If, \( k > 1 \),
\[ f(k) = k^2 \text{ and } \lim_{x \to k} f(x) = \lim_{x \to k} (x^2) = k^2. \]
Here, \( \lim_{x \to k} f(x) = f(k) \)
Hence, the function \( f \) is continuous for all real values greater than 1.
Hence, the function \( f \) is discontinuous only at \( x = 1 \).

**Question 13:**

Is the function defined by \( f(x) = \begin{cases} 
  x + 5, & \text{if } x \leq 1 \\
  x - 5, & \text{if } x > 1
\end{cases} \) a continuous function?

**Answer 13:**
Let, \( k \) be any real number. According to question, \( k < 1 \) or \( k = 1 \) or \( k > 1 \)

First case: If, \( k < 1 \),
\[ f(k) = k + 5 \text{ and } \lim_{x \to k^-} f(x) = \lim_{x \to k^-} (x + 5) = k + 5. \]
Here, \( \lim_{x \to k} f(x) = f(k) \)
Hence, the function \( f \) is continuous for all real numbers less than 1.

Second case: If, \( k = 1 \), \( f(1) = 1 + 5 = 6 \)
LHL = \( \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x + 5) = 6 \)
RHL = \( \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x - 5) = -4, \)
Here, at \( x = 1 \), LHL \( \neq \) RHL. Hence, the function \( f \) is discontinuous at \( x = 1 \).

Third case: If, \( k > 1 \),
\[ f(k) = k - 5 \text{ and } \lim_{x \to k^-} f(x) = \lim_{x \to k^-} (x - 5) = k - 5. \]
Here, \( \lim_{x \to k} f(x) = f(k) \)
Hence, the function \( f \) is continuous for all real numbers greater than 1.
Hence, the function \( f \) is discontinuous only at \( x = 1 \).

Discuss the continuity of the function \( f \), where \( f \) is defined by:
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**Question 14:**

\[
f(x) = \begin{cases} 
3, & \text{if } 0 \leq x \leq 1 \\
4, & \text{if } 1 < x < 3 \\
5, & \text{if } 3 \leq x \leq 10
\end{cases}
\]

**Answer 14:**

Let, \(k\) be any real number. According to question, 
\(0 \leq k \leq 1\) or \(k = 1\) or \(1 < k < 3\) or \(k = 3\) or \(3 \leq k \leq 10\)

**First case:** If, \(0 \leq k \leq 1\),
\(f(k) = 3\) and \(\lim_{x \to k} f(x) = \lim_{x \to k} (3) = 3\). Here, \(\lim_{x \to k} f(x) = f(k)\)
Hence, the function \(f\) is continuous for \(0 \leq x \leq 1\).

**Second case:** If, \(k = 1\), \(f(1) = 3\)
LHL = \(\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (3) = 3\)
RHL = \(\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (4) = 4\),
Here, at \(x = 1\), LHL \(\neq\) RHL. Hence, the function \(f\) is discontinuous at \(x = 1\).

**Third case:** If, \(1 < k < 3\),
\(f(k) = 4\) and \(\lim_{x \to k} f(x) = \lim_{x \to k} (4) = 4\). Here, \(\lim_{x \to k} f(x) = f(k)\)
Hence, the function \(f\) is continuous for \(1 < x < 3\).

**Fourth case:** If \(k = 3\),
LHL = \(\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (4) = 4\) and \(\) RHL = \(\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (5) = 5\),
Here, at \(x = 3\), LHL \(\neq\) RHL. Hence, the function \(f\) is discontinuous at \(x = 3\).

**Fifth case:** If, \(3 \leq k \leq 10\),
\(f(k) = 5\) and \(\lim_{x \to k} f(x) = \lim_{x \to k} (5) = 5\). Here, \(\lim_{x \to k} f(x) = f(k)\)
Hence, the function \(f\) is continuous for \(3 \leq x \leq 10\).
Hence, the function \(f\) is discontinuous only at \(x = 1\) and \(x = 3\).

**Question 15:**

\[
f(x) = \begin{cases} 
2x, & \text{if } x < 0 \\
0, & \text{if } 0 \leq x \leq 1 \\
4x, & \text{if } x > 1
\end{cases}
\]

**Answer 15:**

Let, \(k\) be any real number. According to question, 
\(k < 0\) or \(k = 0\) or \(0 \leq k \leq 1\) or \(k = 1\) or \(k > 1\)

**First case:** If, \(k < 0\),
\(f(k) = 2k\) and \(\lim_{x \to k} f(x) = \lim_{x \to k} (2x) = 2k\). Here, \(\lim_{x \to k} f(x) = f(k)\)
Hence, the function \(f\) is continuous for all real numbers less than 0.

**Second case:** If, \(k = 0\), \(f(0) = 0\)
LHL = \(\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (2x) = 0\)
RHL = \(\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (0) = 0\). Here, \(\lim_{x \to k} f(x) = f(k)\)
Hence, the function \(f\) is continuous at \(x = 0\).

**Third case:** If, \(0 \leq k \leq 1\),
\(f(k) = 0\) and \(\lim_{x \to k} f(x) = \lim_{x \to k} (0) = 0\). Here, \(\lim_{x \to k} f(x) = f(k)\)
Hence, the function $f$ is continuous at $0 \leq x \leq 1$.

**Fourth case:** If $k = 1$,
\[
\text{LHL } = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (2x) = 2
\]
\[
\text{RHL } = \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (2x) = 2
\]
Here, at $x = 1$, LHL $\neq$ RHL.
Hence, the function $f$ is discontinuous at $x = 1$.

**Fifth case:** If $k > 1$,
\[
f(k) = 2k \quad \text{and} \quad \lim_{x \to k} f(x) = \lim_{x \to k} (2x) = 2k
\]
Here, $\lim_{x \to k} f(x) = f(k)$

Hence, the function $f$ is continuous for all real numbers greater than 1. Therefore, the function $f$ is discontinuous only at $x = 1$.

**Question 16:**
\[
f(x) = \begin{cases} 
-2, & \text{if } x \leq -1 \\
2x, & \text{if } -1 < x \leq 1 \\
2, & \text{if } x > 1
\end{cases}
\]

**Answer 16:**
Let, $k$ be any real number.
According to question, $k < -1$ or $k = -1$ or $-1 < x \leq 1$ or $k = 1$ or $k > 1$

**First case:** If, $k < -1$,
\[
f(k) = -2 \quad \text{and} \quad \lim_{x \to k} f(x) = \lim_{x \to k} (-2) = -2
\]
Here, $\lim_{x \to k} f(x) = f(k)$
Hence, the function $f$ is continuous for all real numbers less than $-1$.

**Second case:** If, $k = -1$,
\[
f(-1) = -2 \quad \text{and} \quad \lim_{x \to -1} f(x) = \lim_{x \to -1} (-2) = -2
\]
Here, $\lim_{x \to -1} f(x) = f(k)$
Hence, the function $f$ is continuous at $x = -1$.

**Third case:** If, $-1 < x \leq 1$,
\[
f(k) = 2k \quad \text{and} \quad \lim_{x \to k} f(x) = \lim_{x \to k} (2x) = 2k
\]
Here, $\lim_{x \to k} f(x) = f(k)$
Hence, the function $f$ is continuous at $-1 < x \leq 1$.

**Fourth case:** If, $k = 1$,
\[
\text{LHL } = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (2x) = 2
\]
\[
\text{RHL } = \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (2x) = 2
\]
Here, $\lim_{x \to 1^-} f(x) = f(k)$
Hence, the function $f$ is continuous at $x = 1$.

**Fifth case:** If, $k > 1$,
\[
f(k) = 2 \quad \text{and} \quad \lim_{x \to k} f(x) = \lim_{x \to k} (2) = 2
\]
Here, $\lim_{x \to k} f(x) = f(k)$
Hence, the function $f$ is continuous for all real numbers greater than 1. Therefore, the function $f$ is continuous for all real numbers.
Question 17:
Find the relationship between \(a\) and \(b\) so that the function \(f\) defined by
\[
f(x) = \begin{cases} 
ax + 1, & \text{if } x \leq 3 \\
bx + 3, & \text{if } x > 3 
\end{cases}
\]
is continuous at \(x = 3\).

Answer 17:
Given that the function is continuous at \(x = 3\). Therefore, \(LHL = RHL = f(3)\)
\[
\Rightarrow \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)
\]
\[
\Rightarrow \lim_{x \to 3^-} ax + 1 = \lim_{x \to 3^+} bx + 3 = 3a + 1
\]
\[
\Rightarrow 3a + 1 = 3b + 3 = 3a + 1
\]
\[
\Rightarrow 3a = 3b + 2 \quad \Rightarrow a = b + \frac{2}{3}
\]

Question 18:
For what value of \(\lambda\) is the function defined by
\[
f(x) = \begin{cases} 
\lambda(x^2 - 2x), & \text{if } x \leq 0 \\
4x + 1, & \text{if } x > 0 
\end{cases}
\]
continuous at \(x = 0\)? What about continuity at \(x = 1\)?

Answer 18:
Given that the function is continuous at \(x = 0\). Therefore, \(LHL = RHL = f(0)\)
\[
\Rightarrow \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)
\]
\[
\Rightarrow \lim_{x \to 0^-} \lambda(x^2 - 2x) = \lim_{x \to 0^+} 4x + 1 = \lambda[(0)^2 - 2(0)]
\]
\[
\Rightarrow \lambda[(0)^2 - 2(0)] = 4(0) + 1 = \lambda(0)
\]
\[
\Rightarrow 0 = \lambda + 1 \quad \Rightarrow \lambda = \frac{1}{0}
\]
Hence, there is no real value of \(\lambda\) for which the given function be continuous.
If, \(x = 1\),
\[
f(1) = 4(1) + 1 = 5
\]
and \(\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1) = 5\), Here, \(\lim f(x) = f(1)\)
Hence, the function \(f\) is continuous for all real values of \(\lambda\).

Question 19:
Show that the function defined by \(g(x) = x - [x]\) is discontinuous at all integral points. Here \([x]\) denotes the greatest integer less than or equal to \(x\).

Answer 19:
Let, \(k\) be any integer.
\(LHL = \lim_{x \to k^-} f(x) = \lim_{x \to k^-} x - [x] = k - (k - 1) = 1\)
\(RHL = \lim_{x \to k^+} f(x) = \lim_{x \to k^+} x - [x] = k - (k) = 0\),
Hence, at \(x = k\), \(LHL \neq RHL\). Hence, the function \(f\) is discontinuous for all integers.

Question 20:
Is the function defined by \(f(x) = x^2 - \sin x + 5\) continuous at \(x = \pi\).

Answer 20:
Given function: \(f(x) = x^2 - \sin x + 5\),
At \(x = \pi\), \(f(\pi) = \pi^2 - \sin \pi + 5 = \pi^2 - 0 + 5 = \pi^2 + 5\)

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\[
l\lim_{x \to n} f(x) = \lim_{x \to n} x^2 - \sin x + 5 = \pi^2 - \sin \pi + 5 = \pi^2 - 0 + 5 = \pi^2 + 5
\]

Here, at \( x = \pi \), \( \lim_{x \to \pi} f(x) = f(\pi) = \pi^2 + 5 \)

Hence, the function \( f \) is continuous at \( x = \pi \).

**Question 21:**
Discuss the continuity of the following functions:

(a) \( f(x) = \sin x + \cos x \)
(b) \( f(x) = \sin x - \cos x \)
(c) \( f(x) = \sin x \cdot \cos x \)

**Answer 21:**
Let, \( g(x) = \sin x \)
Let, \( k \) be any real number. At \( x = k \), \( g(k) = \sin k \)
LHL = \( \lim_{x \to k^-} g(x) = \lim_{x \to k^-} \sin x = \lim_{h \to 0} \sin(k - h) = \lim_{h \to 0} \sin k \cos h - \cos k \sin h = \sin k \)
RHL = \( \lim_{x \to k^+} g(x) = \lim_{x \to k^+} \sin x = \lim_{h \to 0} \sin(k + h) = \lim_{h \to 0} \sin k \cos h + \cos k \sin h = \sin k \)
Here, at \( x = k \), LHL = RHL = \( g(k) \).
Hence, the function \( g \) is continuous for all real numbers.

Let, \( h(x) = \cos x \)
Let, \( k \) be any real number. \( x = k \) पार, \( h(k) = \cos k \)
LHL = \( \lim_{x \to k^-} h(x) = \lim_{x \to k^-} \cos x = \lim_{h \to 0} \cos(k - h) = \lim_{h \to 0} \cos k \cos h + \sin k \sin h = \cos k \)
RHL = \( \lim_{x \to k^+} h(x) = \lim_{x \to k^+} \cos x = \lim_{h \to 0} \cos(k + h) = \lim_{h \to 0} \cos k \cos h - \sin k \sin h = \cos k \)
Here, at \( x = k \), LHL = RHL = \( h(k) \).
Hence, the function \( h \) is continuous for all real numbers.

We know that if \( g \) and \( h \) are two continuous functions, then the functions \( g + h, g - h \) and \( gh \) also be a continuous functions.
Hence, (a) \( f(x) = \sin x + \cos x \) (b) \( f(x) = \sin x - \cos x \) and (c) \( f(x) = \sin x \cdot \cos x \) are continuous functions.

**Question 22:**
Discuss the continuity of the cosine, cosecant, secant and cotangent functions.

**Answer 22:**
Let \( g(x) = \sin x \)
Let, \( k \) be any real number. At \( x = k \), \( g(k) = \sin k \)
LHL = \( \lim_{x \to k^-} g(x) = \lim_{x \to k^-} \sin x = \lim_{h \to 0} \sin(k - h) = \lim_{h \to 0} \sin k \cos h - \cos k \sin h = \sin k \)
RHL = \( \lim_{x \to k^+} g(x) = \lim_{x \to k^+} \sin x = \lim_{h \to 0} \sin(k + h) = \lim_{h \to 0} \sin k \cos h + \cos k \sin h = \sin k \)
Here, at \( x = k \), LHL = RHL = \( g(k) \).
Hence, the function \( g \) is continuous for all real numbers.

Let \( h(x) = \cos x \)
Let, \( k \) be any real number. At \( x = k \), \( h(k) = \cos k \)
LHL = \( \lim_{x \to k^-} h(x) = \lim_{x \to k^-} \cos x = \lim_{h \to 0} \cos(k - h) = \lim_{h \to 0} \cos k \cos h + \sin k \sin h = \cos k \)
RHL = \( \lim_{x \to k^+} h(x) = \lim_{x \to k^+} \cos x = \lim_{h \to 0} \cos(k + h) = \lim_{h \to 0} \cos k \cos h - \sin k \sin h = \cos k \)
Here, at \( x = k \), LHL = RHL = \( h(k) \).
Hence, the function \( h \) is continuous for all real numbers.

We know that if \( g \) and \( h \) are two continuous functions, then the functions \( g, h \neq 0 \), \( \frac{1}{h} \), \( h \neq 0 \) and \( \frac{1}{g}, g \neq 0 \) be continuous functions.
Therefore, \( \csc x = \frac{1}{\sin x} \), \( \sin x \neq 0 \) is continuous \( \Rightarrow x \neq n\pi \ (n \in \mathbb{Z}) \) is continuous.
Hence, \( \csc x \) is continuous except \( x = n\pi \ (n \in \mathbb{Z}) \).

\[ \sec x = \frac{1}{\cos x}, \cos x \neq 0 \text{ is continuous.} \Rightarrow x \neq \frac{(2n+1)\pi}{2} \ (n \in \mathbb{Z}) \text{ is continuous.} \]
Hence, \( \sec x \) is continuous except \( x = \frac{(2n+1)\pi}{2} \ (n \in \mathbb{Z}) \).

\[ \cot x = \frac{\cos x}{\sin x}, \sin x \neq 0 \text{ is continuous.} \Rightarrow x \neq n\pi \ (n \in \mathbb{Z}) \text{ is continuous.} \]
Hence, \( \cot x \) is continuous except \( x = n\pi \ (n \in \mathbb{Z}) \).

**Question 23:**
Find all points of discontinuity of \( f \), where
\[
f(x) = \begin{cases} 
\frac{\sin x}{x}, & \text{if } x < 0 \\
x + 1, & \text{if } x \geq 0
\end{cases}
\]

**Answer 23:**
Let, \( k \) be any real number. According to question, \( k < 0 \) or \( k = 0 \) or \( k > 0 \)
- **First case:** If, \( k < 0 \),
  \[ f(k) = \frac{\sin k}{k} \text{ and } \lim_{x \to k} f(x) = \lim_{x \to k} \left( \frac{\sin x}{x} \right) = \frac{\sin k}{k}. \] Here, \( \lim f(x) = f(k) \)
  Hence, the function \( f \) is continuous for all real numbers less than 0.
- **Second case:** If, \( k = 0 \), \( f(0) = 0 + 1 = 1 \)
  \[ LHL = \lim_{x \to 0^-} f(x) = \lim_{x \to 0} (x + 1) = 0 + 1 = 1 \]
  \[ RHL = \lim_{x \to 0^+} f(x) = \lim_{x \to 0} (x + 1) = 0 + 1 = 1, \] Here, at \( x = 0 \), \( LHL = RHL = f(0) \). Hence, the function \( f \) is continuous at \( x = 0 \).
- **Third case:** If, \( k > 0 \),
  \[ f(k) = k + 1 \text{ and } \lim_{x \to k} f(x) = \lim_{x \to k} (x + 1) = k + 1, \] Here, \( \lim f(x) = f(k) \)
  Hence, the function \( f \) is continuous for all real numbers greater than 0.
  Therefore, the function \( f \) is continuous for all real numbers.

**Question 24:**
Determine if \( f \) defined by
\[
f(x) = \begin{cases} 
x^2 \sin \frac{1}{x}, & \text{if } x \neq 0 \\
0, & \text{if } x = 0
\end{cases}
\]
is a continuous function?

**Answer 24:**
Let, \( k \) be any real number. According to question, \( k \neq 0 \) or \( k = 0 \)
- **First case:** If, \( k \neq 0 \),
  \[ f(k) = k^2 \sin \frac{1}{k} \text{ and } \lim_{x \to k} f(x) = \lim_{x \to k} \left( x^2 \sin \frac{1}{x} \right) = k^2 \sin \frac{1}{k}. \] Here, \( \lim f(x) = f(k) \)
  Hence, the function \( f \) is continuous for \( k \neq 0 \).
- **Second case:** If, \( k = 0 \), \( f(0) = 0 \)
  \[ LHL = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \left( x^2 \sin \frac{1}{x} \right) \]
  \[ RHL = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \left( x^2 \sin \frac{1}{x} \right) \]
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We know that, $-1 \leq \sin \frac{1}{x} \leq 1$, $x \neq 0 \quad \Rightarrow -x^2 \leq \sin \frac{1}{x} \leq x^2$

$\Rightarrow \lim_{x \to 0}(-x^2) \leq \lim_{x \to 0} \frac{1}{x} \leq \lim_{x \to 0} x^2$

$\Rightarrow 0 \leq \lim_{x \to 0} \sin \frac{1}{x} \leq 0 \quad \Rightarrow \lim_{x \to 0} \sin \frac{1}{x} = 0 \quad \Rightarrow \lim_{x \to 0} x^2 \sin \frac{1}{x} = 0 \quad \Rightarrow \lim_{x \to 0} f(x) = 0$

Similarly, RHL = $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \left(x^2 \sin \frac{1}{x}\right) = \lim_{x \to 0} \left(x^2 \sin \frac{1}{x}\right) = 0$,

Here, at $x = 0$, LHL = RHL = $f(0)$

Hence, at $x = 0$, $f$ is continuous.

Hence, the function $f$ is continuous for all real numbers.

**Question 25:**

Examine the continuity of $f$, where $f$ is defined by

$$f(x) = \begin{cases} \sin x - \cos x, & \text{if } x \neq 0 \\ -1, & \text{if } x = 0 \end{cases}$$

**Answer 25:**

Let, $k$ be any real number. According to question, $k \neq 0$ or $k = 0$

**First case:** If, $k \neq 0$, $f(0) = 0 - 1 = -1$

LHL = $\lim_{k \to 0^+} f(x) = \lim_{k \to 0^+} (\sin x - \cos x) = 0 - 1 = -1$

RHL = $\lim_{k \to 0^+} f(x) = \lim_{k \to 0^+} (\sin x - \cos x) = 0 - 1 = -1$

Hence, at $x \neq 0$, LHL = RHL = $f(x)$

Hence, the function $f$ is continuous at $x \neq 0$.

**Second case:** If, $k = 0$, $f(k) = -1$

and $\lim_{x \to k} f(x) = \lim_{x \to k} (-1) = -1$, Here, $\lim f(x) = f(k)$

Hence, the function $f$ is continuous at $x = 0$.

Therefore, the function $f$ is continuous for all real numbers.

Find the values of $k$ so that the function $f$ is continuous at the indicated point in exercises 26 to 29.

**Question 26:**

$$f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases} \quad \text{at } x = \frac{\pi}{2}$$

**Answer 26:**

Given that the function is continuous at $x = \frac{\pi}{2}$. Therefore, LHL = RHL = $f\left(\frac{\pi}{2}\right)$

$\Rightarrow \lim_{x \to \frac{\pi}{2}} f(x) = \lim_{x \to \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$

$\Rightarrow \lim_{x \to \frac{\pi}{2}} \frac{k \cos x}{\pi - 2x} = \lim_{x \to \frac{\pi}{2}} \frac{k \cos x}{\pi - 2x} = 3$

$\Rightarrow \lim_{h \to 0} \frac{k \cos \left(\frac{\pi}{2} - h\right)}{\pi - 2 \left(\frac{\pi}{2} - h\right)} = \lim_{h \to 0} \frac{k \cos \left(\frac{\pi}{2} + h\right)}{\pi - 2 \left(\frac{\pi}{2} + h\right)} = 3$
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\[ \lim_{h \to 0} \frac{k \sin h}{2h} = \lim_{h \to 0} \frac{-k \sin h}{-2h} = 3 \]
\[ \frac{k}{2} \cdot \frac{k}{2} = 3 \]
\[ k = 6 \]

**Question 27:**

\[ f(x) = \begin{cases} 
  kx^2, & \text{if } x \leq 2 \\
  3, & \text{if } x > 2 
\end{cases} \text{ at } x = 2 \]

**Answer 27:**

Given that the function is continuous at \( x = 2 \).
Therefore, LHL = RHL = \( f(2) \)
\[ \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2) \]
\[ \lim_{x \to 2^-} kx^2 = \lim_{x \to 2^+} 3 = k(2)^2 \]
\[ 4k = 3 = 4k \]
\[ k = \frac{3}{4} \]

**Question 28:**

\[ f(x) = \begin{cases} 
  kx + 1, & \text{if } x \leq \pi \\
  \cos x, & \text{if } x > \pi 
\end{cases} \text{ at } x = \pi \]

**Answer 28:**

Given that the function is continuous at \( x = \pi \).
Therefore, LHL = RHL = \( f(\pi) \)
\[ \lim_{x \to \pi^-} f(x) = \lim_{x \to \pi^+} f(x) = f(\pi) \]
\[ \lim_{x \to \pi^-} kx + 1 = \lim_{x \to \pi^+} \cos x = k(\pi) + 1 \]
\[ k(\pi) + 1 = \cos \pi = k\pi + 1 \]
\[ k\pi + 1 = -1 = k\pi + 1 \]
\[ \pi k = -2 \]
\[ k = -\frac{2}{\pi} \]

**Question 29:**

\[ f(x) = \begin{cases} 
  kx + 1, & \text{if } x \leq 5 \\
  3x - 5, & \text{if } x > 5 
\end{cases} \text{ at } x = 5 \]

**Answer 29:**

Given that the function is continuous at \( x = 5 \).
Therefore, LHL = RHL = \( f(5) \)
\[ \lim_{x \to 5^-} f(x) = \lim_{x \to 5^+} f(x) = f(5) \]
\[ \lim_{x \to 5^-} kx + 1 = \lim_{x \to 5^+} 3x - 5 = 5k + 1 \]
\[ 5k + 1 = 15 - 5 = 5k + 1 \]
\[ 5k = 9 \]
\[ k = \frac{9}{5} \]
Question 30:
Find the values of \( a \) and \( b \) such that the function defined by
\[
f(x) = \begin{cases} 
5, & \text{if } x \leq 2 \\
ax + b, & \text{if } 2 < x < 10 \\
21, & \text{if } x \geq 10
\end{cases}
\]
is a continuous function.

Answer 30:
Given that the function is continuous at \( x = 2 \). Therefore, \( \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2) \)
\[
\Rightarrow \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = 5
\]
\[
\Rightarrow 2a + b = 5 \quad \text{(1)}
\]
Given that the function is continuous at \( x = 10 \). Therefore, \( \lim_{x \to 10^-} f(x) = \lim_{x \to 10^+} f(x) = f(10) \)
\[
\Rightarrow \lim_{x \to 10^-} f(x) = \lim_{x \to 10^+} f(x) = 21
\]
\[
\Rightarrow 10a + b = 21 \quad \text{(2)}
\]
Solving the equation (1) and (2), we get
\[
a = 2 \quad b = 1
\]

Question 31:
Show that the function defined by \( f(x) = \cos(x^2) \) is a continuous function.

Answer 31:
Assuming that the functions are well defined for all real numbers, we can write the given function \( f \) in the combination of \( g \) and \( h \) \((f = gh)\). Where, \( g(x) = \cos x \) and \( h(x) = x^2 \). If \( g \) and \( h \) both are continuous function then \( f \) also be continuous.
\[
(\because \text{goh}(x) = g(h(x)) = g(x^2) = \cos(x^2))
\]
Function \( g(x) = \cos x \)
Let, \( k \) be any real number. At \( x = k \), \( g(k) = \cos k \)
\[
\lim_{x \to k} g(x) = \lim_{x \to k} \cos x = \lim_{h \to 0} \cos(k + h) = \lim_{h \to 0} \cos k \cos h - \sin k \sin h = \cos k
\]
Here, \( \lim_{x \to k} g(x) = g(k) \). Hence, the function \( g \) is continuous for all real numbers.

Function \( h(x) = x^2 \)
Let, \( k \) be any real number. At \( x = k \), \( h(k) = k^2 \)
\[
\lim_{x \to k} h(x) = \lim_{x \to k} x^2 = k^2
\]
Here, \( \lim_{x \to k} h(x) = h(k) \). Hence, the function \( h \) is continuous for all real numbers.
Therefore, \( g \) and \( h \) both are continuous function. Hence, \( f \) is continuous.

Question 32:
Show that the function defined by \( f(x) = |\cos x| \) is a continuous function.

Answer 32:
Assuming that the functions are well defined for all real numbers, we can write the given function \( f \) in the combination of \( g \) and \( h \) \((f = gh)\). Where, \( g(x) = |x| \) and \( h(x) = \cos x \). If \( g \) and \( h \) both are continuous function then \( f \) also be continuous.
\[
(\because \text{goh}(x) = g(h(x)) = g(\cos x) = |\cos x|)
\]
Function \( g(x) = |x| \)
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Rearranging the function \( g \), we get
\[
g(x) = \begin{cases} 
-x, & \text{if } x < 0 \\
(x, & \text{if } x \geq 0 
\end{cases} 
\]

Let, \( k \) be any real number. According to question, \( k < 0 \) or \( k = 0 \) or \( k > 0 \)

**First case:** If, \( k < 0 \),
\[
g(k) = 0 \quad \text{and} \quad \lim_{x \to k} g(x) = \lim_{x \to k} (-x) = 0, \quad \text{here, } \lim_{x \to k} g(x) = g(k)
\]
Hence, the function \( g \) is continuous for all real numbers less than 0.

**Second case:** If, \( k = 0 \), \( g(0) = 0 + 1 = 1 \)
\[
\text{LHL} = \lim_{x \to 0^-} g(x) = \lim_{x \to 0^-} (-x) = 0 \\
\text{RHL} = \lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} (x) = 0,
\]
Here, at \( x = 0 \), LHL = RHL = \( g(0) \)
Hence, the function \( g \) is continuous at \( x = 0 \).

**Third case:** If, \( k > 0 \),
\[
g(k) = 0 \quad \text{and} \quad \lim_{x \to k} g(x) = \lim_{x \to k} (x) = 0, \quad \text{Here, } \lim_{x \to k} g(x) = g(k)
\]
Hence, the function \( g \) is continuous for all real numbers greater than 0.
Hence, the function \( g \) is continuous for all real numbers.

Function \( h(x) = \cos x \)
Let, \( k \) be any real number. At \( x = k \), \( h(k) = \cos k \)
\[
\lim_{x \to k} h(x) = \lim_{x \to k} \cos x = \cos k
\]
Here, \( \lim_{x \to k} h(x) = h(k) \), Hence, the function \( h \) is continuous for all real numbers.
Therefore, \( g \) and \( h \) both are continuous function. Hence, \( f \) is continuous.

**Question 33:**
Examine that \( \sin |x| \) is a continuous function.

**Answer 33:**
Assuming that the functions are well defined for all real numbers, we can write the given function \( f \) in the combination of \( g \) and \( h \) (\( f = h \circ g \)). Where, \( h(x) = \sin x \) and \( g(x) = |x| \). If \( g \) and \( h \) both are continuous function then \( f \) also be continuous.
\[
[:: h \circ g(x) = h(g(x)) = h(|x|) = \sin |x|]
\]
Function \( h(x) = \sin x \)
Let, \( k \) be any real number. At \( x = k \), \( h(k) = \sin k \)
\[
\lim_{x \to k} h(x) = \lim_{x \to k} \sin x = \sin k
\]
Here, \( \lim_{x \to k} h(x) = h(k) \), Hence, the function \( h \) is continuous for all real numbers.
Function \( g(x) = |x| \)
Redefining the function \( g \), we get
\[
g(x) = \begin{cases} 
-x, & \text{if } x < 0 \\
(x, & \text{if } x \geq 0 
\end{cases} 
\]
Let, \( k \) be any real number. According to question, \( k < 0 \) or \( k = 0 \) or \( k > 0 \)

**First case:** If, \( k < 0 \),
\[
g(k) = 0 \quad \text{and} \quad \lim_{x \to k} g(x) = \lim_{x \to k} (-x) = 0, \quad \text{Here, } \lim_{x \to k} g(x) = g(k)
\]
Hence, the function \( g \) is continuous for all real numbers less than 0.
Second case: If, \( k = 0 \), \( g(0) = 0 + 1 = 1 \)
LHL = \( \lim_{x \to 0^-} g(x) = \lim_{x \to 0^-} (-x) = 0 \)  
RHL = \( \lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} (x) = 0 \)
Here, at \( x = 0 \), LHL = RHL = \( g(0) \)
Hence, at \( x = 0 \), the function \( g \) is continuous.

Third case: If, \( k > 0 \),
\( g(k) = 0 \) and \( \lim_{x \to k} g(x) = \lim_{x \to k} (x) = 0 \), Here, \( \lim g(x) = g(k) \)
Hence, the function \( g \) is continuous for all real numbers greater than 0.
Hence, the function \( g \) is continuous for all real numbers.
Therefore, \( g \) and \( h \) both are continuous function. Hence, \( f \) is continuous.

**Question 34:**
Find all the points of discontinuity of \( f \) defined by \( f(x) = |x| - |x + 1| \).

**Answer 34:**
Assuming that the functions are well defined for all real numbers, we can write the given function \( f \) in the combination of \( g \) and \( h \) (\( f = g - h \)), where, \( g(x) = |x| \) and \( h(x) = |x + 1| \). If \( g \) and \( h \) both are continuous function then \( f \) also be continuous.
Function \( g(x) = |x| \)
Redefining the function \( g \), we get,
\[
g(x) = \begin{cases} 
-x, & \text{if } x < 0 \\
x, & \text{if } x \geq 0 
\end{cases}
\]
Let, \( k \) be any real number. According to question, \( k < 0 \) or \( k = 0 \) or \( k > 0 \)

First case: If, \( k < 0 \),
\( g(k) = 0 \) and \( \lim_{x \to k} g(x) = \lim_{x \to k} (-x) = 0 \), Here, \( \lim g(x) = g(k) \)
Hence, the function \( g \) is continuous for all real numbers less than 0.

Second case: If, \( k = 0 \), \( g(0) = 0 + 1 = 1 \)
LHL = \( \lim_{x \to 0^-} g(x) = \lim_{x \to 0^-} (-x) = 0 \) and \( \text{RHL} = \lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} (x) = 0 \)
Here, at \( x = 0 \), LHL = RHL = \( g(0) \)
Hence, the function \( g \) is continuous at \( x = 0 \).

Third case: If, \( k > 0 \),
\( g(k) = 0 \) and \( \lim_{x \to k} g(x) = \lim_{x \to k} (x) = 0 \), Here, \( \lim g(x) = g(k) \)
Hence, the function \( g \) is continuous for all real numbers more than 0.
Hence, the function \( g \) is continuous for all real numbers.

Function \( h(x) = |x + 1| \)
Redefining the function \( h \), we get
\[
h(x) = \begin{cases} 
-(x + 1), & \text{if } x < -1 \\
x + 1, & \text{if } x \geq -1 
\end{cases}
\]
Let, \( k \) be any real number. According to question, \( k < -1 \) or \( k = -1 \) or \( k > -1 \)

First case: If, \( k < -1 \),
\( h(k) = -(k + 1) \) and \( \lim_{x \to k} h(x) = \lim_{x \to k} (-k + 1) = -(k + 1) \), Here, \( \lim h(x) = h(k) \)
Hence, the function \( g \) is continuous for all real numbers less than \(-1\).

Second case: If, \( k = -1 \), \( h(-1) = -1 + 1 = 0 \)
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LHL = \lim_{x \to -1^-} h(x) = \lim_{x \to -1^-} (-1 + 1) = 0

RHL = \lim_{x \to -1^+} h(x) = \lim_{x \to -1^+} (x + 1) = -1 + 1 = 0,

Here, at \( x = -1 \), LHL = RHL = \( h(-1) \)
Hence, the function \( h \) is continuous at \( x = -1 \).

Third case: If, \( k > -1 \),
\( h(k) = k + 1 \) and \( \lim_{x \to k} h(x) = \lim_{x \to k} (k + 1) = k + 1 \), Here, \( \lim h(x) = h(k) \)

Hence, the function \( g \) is continuous for all real numbers greater than \(-1\).
Hence, the function \( h \) is continuous for all real numbers.

Therefore, \( g \) and \( h \) both are continuous function. Hence, \( f \) is continuous.