

Mathematics

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(Chapter - 5) (Continuity and Differentiability)

(Class 12)

Exercise 5.4

Differentiate the following w.r.t. x :

Question 1:

$$\frac{e^x}{\sin x}$$

Answer 1:

Let $y = \frac{e^x}{\sin x}$, Therefore,

$$\frac{dy}{dx} = \frac{e^x \cdot \frac{d}{dx} \sin x - \sin x \cdot \frac{d}{dx} e^x}{\sin^2 x} = \frac{e^x \cdot \cos x - \sin x \cdot e^x}{\sin^2 x} = \frac{e^x(\cos x - \sin x)}{\sin^2 x}$$

Question 2:

$$e^{\sin^{-1} x}$$

Answer 2:

Let $y = e^{\sin^{-1} x}$, Therefore,

$$\frac{dy}{dx} = e^{\sin^{-1} x} \cdot \frac{d}{dx} \sin^{-1} x = e^{\sin^{-1} x} \cdot \frac{1}{\sqrt{1-x^2}} = \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}}$$

Question 3:

$$e^{x^3}$$

Answer 3:

Let $y = e^{x^3}$, Therefore,

$$\frac{dy}{dx} = e^{x^3} \cdot \frac{d}{dx} x^3 = e^{x^3} \cdot 3x^2 = 3x^2 e^{x^3}$$

Question 4:

$$\sin(\tan^{-1} e^{-x})$$

Answer 4:

Let $y = \sin(\tan^{-1} e^{-x})$, Therefore,

$$\begin{aligned} \frac{dy}{dx} &= \cos(\tan^{-1} e^{-x}) \cdot \frac{d}{dx} \tan^{-1} e^{-x} = \cos(\tan^{-1} e^{-x}) \cdot \frac{1}{1+(e^{-x})^2} \cdot \frac{d}{dx} e^{-x} \\ &= \cos(\tan^{-1} e^{-x}) \cdot \frac{1}{1+e^{-2x}} \cdot (-e^{-x}) = -\frac{e^{-x} \cos(\tan^{-1} e^{-x})}{1+e^{-2x}} \end{aligned}$$

Question 5:

$$\log(\cos e^x)$$

Answer 5:

Let $y = \log(\cos e^x)$,

Therefore,

$$\frac{dy}{dx} = \frac{1}{\cos e^x} \cdot \frac{d}{dx} \cos e^x = \frac{1}{\cos e^x} (-\sin e^x) \frac{d}{dx} e^x = -\tan e^x \cdot e^x$$

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Question 6:

$$e^x + e^{x^2} + \dots + e^{x^5}$$

Answer 6:

Let $y = e^x + e^{x^2} + e^{x^3} + e^{x^4} + e^{x^5}$, Therefore,

$$\begin{aligned}\frac{dy}{dx} &= e^x + e^{x^2} \frac{d}{dx} x^2 + e^{x^3} \frac{d}{dx} x^3 + e^{x^4} \frac{d}{dx} x^4 + e^{x^5} \frac{d}{dx} x^5 \\ &= e^x + e^{x^2} \cdot 2x + e^{x^3} \cdot 3x^2 + e^{x^4} \cdot 4x^3 + e^{x^5} \cdot 5x^4 \\ &= e^x + 2xe^{x^2} + 3x^2e^{x^3} + 4x^3e^{x^4} + 5x^4e^{x^5}\end{aligned}$$

Question 7:

$$\sqrt{e^{\sqrt{x}}}, x > 0$$

Answer 7:

$$\text{Let } y = \sqrt{e^{\sqrt{x}}}$$

Therefore,

$$\frac{dy}{dx} = \frac{1}{2\sqrt{e^{\sqrt{x}}}} \frac{d}{dx} e^{\sqrt{x}} = \frac{1}{2\sqrt{e^{\sqrt{x}}}} \cdot e^{\sqrt{x}} \cdot \frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{e^{\sqrt{x}}}} \cdot e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{\sqrt{e^{\sqrt{x}}}}{4\sqrt{x}}$$

Question 8:

$$\log(\log x), x > 1$$

Answer 8:

$$\text{Let } y = \frac{e^x}{\sin x}$$

Therefore,

$$\frac{dy}{dx} = \frac{1}{\log x} \cdot \frac{d}{dx} \log x = \frac{1}{\log x} \cdot \frac{1}{x} = \frac{1}{x \log x}$$

Question 9:

$$\frac{\cos x}{\log x}, x > 0$$

Answer 9:

$$\text{Let } y = \frac{\cos x}{\log x}$$

$$\text{Therefore, } \frac{dy}{dx} = \frac{\log x \frac{d}{dx} \cos x - \cos x \frac{d}{dx} \log x}{(\log x)^2} = \frac{\log x \cdot (-\sin x) - \cos x \cdot \frac{1}{x}}{(\log x)^2} = \frac{-(x \sin x \log x + \cos x)}{x(\log x)^2}$$

Question 10:

$$\cos(\log x + e^x)$$

Answer 10:

$$\text{Let } y = \cos(\log x + e^x)$$

Therefore,

$$\frac{dy}{dx} = -\sin(\log x + e^x) \cdot \frac{d}{dx} (\log x + e^x) = -\sin(\log x + e^x) \cdot \left(\frac{1}{x} + e^x \right)$$