Question 1:
Prove that $\sqrt{5}$ is irrational.

**Answer 1:**
Let $\sqrt{5}$ be a rational number.
Therefore, we can find two integers $a, b$ ($b \neq 0$) such that $\sqrt{5} = \frac{a}{b}$. Let $a$ and $b$ have a common factor other than 1. Then we can divide them by the common factor, and assume that $a$ and $b$ are co-prime.

\[ a = \sqrt{5}b \]
\[ \Rightarrow a^2 = 5b^2 \]

Therefore, $a^2$ is divisible by 5 and it can be said that $a$ is divisible by 5.
Let $a = 5k$, where $k$ is an integer

\[ (5k)^2 = 5b^2 \]
\[ \Rightarrow 5k^2 = b^2 \]

This means that $b^2$ is divisible by 5 and hence, $b$ is divisible by 5. This implies that $a$ and $b$ have 5 as a common factor. And this is a contradiction to the fact that $a$ and $b$ are co-prime. Hence, $\sqrt{5}$ cannot be expressed as $\frac{p}{q}$ or it can be said that $\sqrt{5}$ is irrational.

Question 2:
Prove that $3 + 2\sqrt{5}$ is irrational.

**Answer 2:**
Let $3 + 2\sqrt{5}$ be rational.
Therefore, we can find two co-prime integers $a, b$ ($b \neq 0$) such that

\[ 3 + 2\sqrt{5} = \frac{a}{b} \]
\[ \Rightarrow 2\sqrt{5} = \frac{a}{b} - 3 \]
\[ \Rightarrow \sqrt{5} = \frac{1}{2} \left( \frac{a}{b} - 3 \right) \]

Since $a$ and $b$ are integers, $\frac{1}{2} \left( \frac{a}{b} - 3 \right)$ will also be rational and therefore, $\sqrt{5}$ is rational. This contradicts the fact that $\sqrt{5}$ is irrational. Hence, our assumption that $3 + 2\sqrt{5}$ is rational is false. Therefore, $3 + 2\sqrt{5}$ is irrational.

Question 3:
Prove that the following are irrationals:

(i) $\frac{1}{\sqrt{2}}$ 
(ii) $7\sqrt{5}$ 
(iii) $6 + \sqrt{2}$

**Answer 3:**

(i) $\frac{1}{\sqrt{2}}$
Let $\frac{1}{\sqrt{2}}$ be rational.
Therefore, we can find two co-prime integers $a, b$ ($b \neq 0$) such that

\[ \frac{1}{\sqrt{2}} = \frac{a}{b} \]

Or
\[ \frac{b}{a} \] is rational as \( a \) and \( b \) are integers.
Therefore, \( \sqrt{2} \) is rational which contradicts the fact that \( \sqrt{2} \) is irrational.
Hence, our assumption is false and \( \frac{1}{\sqrt{2}} \) is irrational.

\textbf{(ii) 7} \sqrt{5} \\
Let \( 7 \sqrt{5} \) is rational.
Therefore, we can find two co-prime integers \( a, b \) (\( b \neq 0 \)) such that
\[ 7 \sqrt{5} = \frac{a}{b} \]
\[ \Rightarrow \sqrt{5} = \frac{a}{7b} \]

\( \frac{a}{7b} \) is rational as \( a \) and \( b \) are integers.
Therefore, \( \sqrt{5} \) should be rational.
This contradicts the fact that \( \sqrt{5} \) is irrational. Therefore, our assumption that \( 7 \sqrt{5} \) is rational is false. Hence, \( 7 \sqrt{5} \) is irrational.

\textbf{(iii) 6} + \sqrt{2} \\
Let \( 6 + \sqrt{2} \) be rational.
Therefore, we can find two co-prime integers \( a, b \) (\( b \neq 0 \)) such that
\[ 6 + \sqrt{2} = \frac{a}{b} \]
\[ \Rightarrow \sqrt{2} = \frac{a}{b} - 6 \]
Since \( a \) and \( b \) are integers, \( \frac{a}{b} - 6 \) is also rational and hence, \( \sqrt{2} \) should be rational. This contradicts the fact that \( \sqrt{2} \) is irrational. Therefore, our assumption is false and hence, \( 6 + \sqrt{2} \) is irrational.