

# Mathematics

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(Chapter – 13) (Limits and Derivatives)

(Class – XI)

## Exercise 13.1

### Question 1:

Evaluate the Given  $\lim_{x \rightarrow 3} x + 3$  limit:

### Answer 1:

$$\lim_{x \rightarrow 3} x + 3 = 3 + 3 = 6$$

### Question 2:

Evaluate the Given limit:  $\lim_{x \rightarrow \pi} \left( x - \frac{22}{7} \right)$

### Answer 2:

$$\lim_{x \rightarrow \pi} \left( x - \frac{22}{7} \right) = \left( \pi - \frac{22}{7} \right)$$

### Question 3:

Evaluate the Given limit:  $\lim_{r \rightarrow 1} \pi r^2$

### Answer 3:

$$\lim_{r \rightarrow 1} \pi r^2 = \pi (1)^2 = \pi$$

### Question 4:

Evaluate the Given limit:  $\lim_{x \rightarrow 4} \frac{4x+3}{x-2}$

### Answer 4:

$$\lim_{x \rightarrow 4} \frac{4x+3}{x-2} = \frac{4(4)+3}{4-2} = \frac{16+3}{2} = \frac{19}{2}$$

### Question 5:

Evaluate the Given limit:  $\lim_{x \rightarrow -1} \frac{x^{10} + x^5 + 1}{x-1}$

### Answer 5:

$$\lim_{x \rightarrow -1} \frac{x^{10} + x^5 + 1}{x-1} = \frac{(-1)^{10} + (-1)^5 + 1}{-1-1} = \frac{1-1+1}{-2} = -\frac{1}{2}$$



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## Question 6:

Evaluate the Given limit:  $\lim_{x \rightarrow 0} \frac{(x+1)^5 - 1}{x}$

## Answer 6:

$$\lim_{x \rightarrow 0} \frac{(x+1)^5 - 1}{x}$$

Put  $x + 1 = y$  so that  $y \rightarrow 1$  as  $x \rightarrow 0$ .

$$\begin{aligned}\text{Accordingly, } \lim_{x \rightarrow 0} \frac{(x+1)^5 - 1}{x} &= \lim_{y \rightarrow 1} \frac{y^5 - 1}{y - 1} \\ &= \lim_{y \rightarrow 1} \frac{y^5 - 1^5}{y - 1}\end{aligned}$$

$$= 5 \cdot 1^{5-1}$$

$$\left[ \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right]$$

$$= 5$$

$$\therefore \lim_{x \rightarrow 0} \frac{(x+1)^5 - 1}{x} = 5$$

## Question 7:

Evaluate the Given limit:  $\lim_{x \rightarrow 2} \frac{3x^2 - x - 10}{x^2 - 4}$

## Answer 7:

At  $x = 2$ , the value of the given rational function takes the form.  $\frac{0}{0}$

$$\begin{aligned}\therefore \lim_{x \rightarrow 2} \frac{3x^2 - x - 10}{x^2 - 4} &= \lim_{x \rightarrow 2} \frac{(x-2)(3x+5)}{(x-2)(x+2)} \\ &= \lim_{x \rightarrow 2} \frac{3x+5}{x+2} \\ &= \frac{3(2)+5}{2+2} \\ &= \frac{11}{4}\end{aligned}$$



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## Question 8:

Evaluate the Given limit:  $\lim_{x \rightarrow 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$

## Answer 8:

At  $x = 2$ , the value of the given rational function takes the form.  $\frac{0}{0}$

$$\begin{aligned}\therefore \lim_{x \rightarrow 3} \frac{x^4 - 81}{2x^2 - 5x - 3} &= \lim_{x \rightarrow 3} \frac{(x-3)(x+3)(x^2+9)}{(x-3)(2x+1)} \\ &= \lim_{x \rightarrow 3} \frac{(x+3)(x^2+9)}{2x+1} \\ &= \frac{(3+3)(3^2+9)}{2(3)+1} \\ &= \frac{6 \times 18}{7} \\ &= \frac{108}{7}\end{aligned}$$

## Question 9:

Evaluate the Given limit:  $\lim_{x \rightarrow 0} \frac{ax+b}{cx+1}$

## Answer 9:

$$\lim_{x \rightarrow 0} \frac{ax+b}{cx+1} = \frac{a(0)+b}{c(0)+1} = b$$

## Question 10:

Evaluate the Given limit:  $\lim_{z \rightarrow 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1}$

## Answer 10:

$$\lim_{z \rightarrow 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1}$$

At  $z = 1$ , the value of the given function takes the form.  $\frac{0}{0}$

Put  $z^{\frac{1}{6}} = x$  so that  $z \rightarrow 1$  as  $x \rightarrow 1$ .



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$$\begin{aligned}\text{Accordingly, } \lim_{z \rightarrow 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1} &= \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} \\ &= \lim_{x \rightarrow 1} \frac{x^2 - 1^2}{x - 1} \\ &= 2 \cdot 1^{2-1} \quad \left[ \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right] \\ &= 2\end{aligned}$$

$$\therefore \lim_{z \rightarrow 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1} = 2$$

**Question 11:**

Evaluate the Given limit:  $\lim_{x \rightarrow 1} \frac{ax^2 + bx + c}{cx^2 + bx + a}, a + b + c \neq 0$

**Answer 11:**

$$\begin{aligned}\lim_{x \rightarrow 1} \frac{ax^2 + bx + c}{cx^2 + bx + a} &= \frac{a(1)^2 + b(1) + c}{c(1)^2 + b(1) + a} \\ &= \frac{a + b + c}{a + b + c} \\ &= 1 \quad [a + b + c \neq 0]\end{aligned}$$

**Question 12:**

Evaluate the Given limit:  $\lim_{x \rightarrow -2} \frac{x - \frac{1}{2}}{x + 2}$

**Answer 12:**

$$\lim_{x \rightarrow -2} \frac{x - \frac{1}{2}}{x + 2}$$

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At  $x = -2$ , the value of the given function takes the form.  $\frac{0}{0}$

$$\begin{aligned}\text{Now, } \lim_{x \rightarrow -2} \frac{\frac{1}{x} + \frac{1}{2}}{x+2} &= \lim_{x \rightarrow -2} \frac{\left(\frac{2+x}{2x}\right)}{x+2} \\ &= \lim_{x \rightarrow -2} \frac{1}{2x} \\ &= \frac{1}{2(-2)} = \frac{-1}{4}\end{aligned}$$

**Question 13:**

Evaluate the Given limit:  $\lim_{x \rightarrow 0} \frac{\sin ax}{bx}$

**Answer 13:**

$$\lim_{x \rightarrow 0} \frac{\sin ax}{bx}$$

At  $x = 0$ , the value of the given function takes the form  $\frac{0}{0}$

$$\begin{aligned}\text{Now, } \lim_{x \rightarrow 0} \frac{\sin ax}{bx} &= \lim_{x \rightarrow 0} \frac{\sin ax}{ax} \times \frac{ax}{bx} \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin ax}{ax}\right) \times \left(\frac{a}{b}\right) \\ &= \frac{a}{b} \lim_{ax \rightarrow 0} \left(\frac{\sin ax}{ax}\right) \quad [x \rightarrow 0 \Rightarrow ax \rightarrow 0] \\ &= \frac{a}{b} \times 1 \quad \left[\lim_{y \rightarrow 0} \frac{\sin y}{y} = 1\right] \\ &= \frac{a}{b}\end{aligned}$$



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## Question 14:

Evaluate the Given limit:  $\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}$ ,  $a, b \neq 0$

## Answer 14:

$$\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}, a, b \neq 0$$

At  $x = 0$ , the value of the given function takes the form  $\frac{0}{0}$

$$\begin{aligned} \text{Now, } \lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx} &= \lim_{x \rightarrow 0} \frac{\left(\frac{\sin ax}{ax}\right) \times ax}{\left(\frac{\sin bx}{bx}\right) \times bx} \\ &= \left(\frac{a}{b}\right) \times \frac{\lim_{ax \rightarrow 0} \left(\frac{\sin ax}{ax}\right)}{\lim_{bx \rightarrow 0} \left(\frac{\sin bx}{bx}\right)} \quad \left[ \begin{array}{l} x \rightarrow 0 \Rightarrow ax \rightarrow 0 \\ \text{and } x \rightarrow 0 \Rightarrow bx \rightarrow 0 \end{array} \right] \\ &= \left(\frac{a}{b}\right) \times \frac{1}{1} \quad \left[ \lim_{y \rightarrow 0} \frac{\sin y}{y} = 1 \right] \\ &= \frac{a}{b} \end{aligned}$$

## Question 15:

Evaluate the Given limit:  $\lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$

## Answer 16:

$$\lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$$

It is seen that  $x \rightarrow \pi \Rightarrow (\pi - x) \rightarrow 0$



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$$\begin{aligned}\therefore \lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)} &= \frac{1}{\pi} \lim_{(\pi - x) \rightarrow 0} \frac{\sin(\pi - x)}{(\pi - x)} \\ &= \frac{1}{\pi} \times 1 \quad \left[ \lim_{y \rightarrow 0} \frac{\sin y}{y} = 1 \right] \\ &= \frac{1}{\pi}\end{aligned}$$

## Question 16:

Evaluate the given limit:

$$\lim_{x \rightarrow 0} \frac{\cos x}{\pi - x}$$

## Answer 16:

$$\lim_{x \rightarrow 0} \frac{\cos x}{\pi - x} = \frac{\cos 0}{\pi - 0} = \frac{1}{\pi}$$

## Question 17:

Evaluate the Given limit:

$$\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1}$$

## Answer 17:

$$\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1}$$

At  $x = 0$ , the value of the given function takes the form.  $\frac{0}{0}$

Now,



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$$\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1} = \lim_{x \rightarrow 0} \frac{1 - 2 \sin^2 x - 1}{1 - 2 \sin^2 \frac{x}{2} - 1} \quad \left[ \cos x = 1 - 2 \sin^2 \frac{x}{2} \right]$$

$$= \lim_{x \rightarrow 0} \frac{\sin^2 x}{\sin^2 \frac{x}{2}} = \lim_{x \rightarrow 0} \frac{\left( \frac{\sin^2 x}{x^2} \right) \times x^2}{\left( \frac{\sin^2 \frac{x}{2}}{\left( \frac{x}{2} \right)^2} \right) \times \frac{x^2}{4}}$$

$$= 4 \frac{\lim_{x \rightarrow 0} \left( \frac{\sin^2 x}{x^2} \right)}{\lim_{x \rightarrow 0} \left( \frac{\sin^2 \frac{x}{2}}{\left( \frac{x}{2} \right)^2} \right)}$$

$$= 4 \frac{\left( \lim_{x \rightarrow 0} \frac{\sin x}{x} \right)^2}{\left( \lim_{\frac{x}{2} \rightarrow 0} \frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2} \quad \left[ x \rightarrow 0 \Rightarrow \frac{x}{2} \rightarrow 0 \right]$$

$$= 4 \frac{1^2}{1^2} \quad \left[ \lim_{y \rightarrow 0} \frac{\sin y}{y} = 1 \right]$$

$$= 4$$

## Question 18:

Evaluate the Given limit:

$$\lim_{x \rightarrow 0} \frac{ax + x \cos x}{b \sin x}$$





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**Answer 18:**

$$\lim_{x \rightarrow 0} \frac{ax + x \cos x}{b \sin x}$$

At  $x = 0$ , the value of the given function takes the form.  $\frac{0}{0}$

Now,

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{ax + x \cos x}{b \sin x} &= \frac{1}{b} \lim_{x \rightarrow 0} \frac{x(a + \cos x)}{\sin x} \\ &= \frac{1}{b} \lim_{x \rightarrow 0} \left( \frac{x}{\sin x} \right) \times \lim_{x \rightarrow 0} (a + \cos x) \\ &= \frac{1}{b} \times \frac{1}{\left( \lim_{x \rightarrow 0} \frac{\sin x}{x} \right)} \times \lim_{x \rightarrow 0} (a + \cos x) \\ &= \frac{1}{b} \times (a + \cos 0) \quad \left[ \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \\ &= \frac{a+1}{b} \end{aligned}$$

**Question 19:**

Evaluate the Given limit:  $\lim_{x \rightarrow 0} x \sec x$

**Answer 19:**

$$\lim_{x \rightarrow 0} x \sec x = \lim_{x \rightarrow 0} \frac{x}{\cos x} = \frac{0}{\cos 0} = \frac{0}{1} = 0$$



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## Question 20:

Evaluate the Given limit:  $\lim_{x \rightarrow 0} \frac{\sin ax + bx}{ax + \sin bx}$   $a, b, a + b \neq 0$

## Answer 20:

At  $x = 0$ , the value of the given function takes the form.  $\frac{0}{0}$

Now,

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\sin ax + bx}{ax + \sin bx} \\ &= \lim_{x \rightarrow 0} \frac{\left(\frac{\sin ax}{ax}\right) ax + bx}{ax + bx \left(\frac{\sin bx}{bx}\right)} \\ &= \frac{\left(\lim_{ax \rightarrow 0} \frac{\sin ax}{ax}\right) \times \lim_{x \rightarrow 0} (ax) + \lim_{x \rightarrow 0} bx}{\lim_{x \rightarrow 0} ax + \lim_{x \rightarrow 0} bx \left(\lim_{bx \rightarrow 0} \frac{\sin bx}{bx}\right)} \quad [\text{As } x \rightarrow 0 \Rightarrow ax \rightarrow 0 \text{ and } bx \rightarrow 0] \\ &= \frac{\lim_{x \rightarrow 0} (ax) + \lim_{x \rightarrow 0} bx}{\lim_{x \rightarrow 0} ax + \lim_{x \rightarrow 0} bx} \quad \left[\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1\right] \\ &= \frac{\lim_{x \rightarrow 0} (ax + bx)}{\lim_{x \rightarrow 0} (ax + bx)} \\ &= \lim_{x \rightarrow 0} (1) \\ &= 1 \end{aligned}$$

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## Question 21:

Evaluate the Given limit:  $\lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x)$

## Answer 21:

At  $x = 0$ , the value of the given function takes the form.  $\infty - \infty$

Now,

$$\begin{aligned} & \lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x) \\ &= \lim_{x \rightarrow 0} \left( \frac{1}{\sin x} - \frac{\cos x}{\sin x} \right) \\ &= \lim_{x \rightarrow 0} \left( \frac{1 - \cos x}{\sin x} \right) \\ &= \lim_{x \rightarrow 0} \left( \frac{1 - \cos x}{x} \right) \left( \frac{x}{\sin x} \right) \\ &= \frac{\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}}{\lim_{x \rightarrow 0} \frac{\sin x}{x}} \\ &= \frac{0}{1} \quad \left[ \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0 \text{ and } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \\ &= 0 \end{aligned}$$

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**Question 22:**

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$$

**Answer 22:**

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$$

At  $x = \frac{\pi}{2}$ , the value of the given function takes the form  $\frac{0}{0}$ .

Now, put  $x - \frac{\pi}{2} = y$  so that  $x \rightarrow \frac{\pi}{2}, y \rightarrow 0$

$$\therefore \lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}} = \lim_{y \rightarrow 0} \frac{\tan 2\left(y + \frac{\pi}{2}\right)}{y}$$

$$= \lim_{y \rightarrow 0} \frac{\tan(\pi + 2y)}{y}$$

$$= \lim_{y \rightarrow 0} \frac{\tan 2y}{y} \quad [\tan(\pi + 2y) = \tan 2y]$$

$$= \lim_{y \rightarrow 0} \frac{\sin 2y}{y \cos 2y}$$

$$= \lim_{y \rightarrow 0} \left( \frac{\sin 2y}{2y} \times \frac{2}{\cos 2y} \right)$$

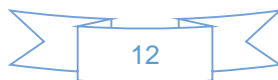
$$= \left( \lim_{2y \rightarrow 0} \frac{\sin 2y}{2y} \right) \times \lim_{y \rightarrow 0} \left( \frac{2}{\cos 2y} \right) \quad [y \rightarrow 0 \Rightarrow 2y \rightarrow 0]$$

$$= 1 \times \frac{2}{\cos 0}$$

$$\left[ \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

$$= 1 \times \frac{2}{1}$$

$$= 2$$



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## Question 23:

Find  $\lim_{x \rightarrow 0} f(x)$  and  $\lim_{x \rightarrow 1} f(x)$ , where  $f(x) = \begin{cases} 2x+3, & x \leq 0 \\ 3(x+1), & x > 0 \end{cases}$

## Answer 23:

The given function is  $f(x) = \begin{cases} 2x+3, & x \leq 0 \\ 3(x+1), & x > 0 \end{cases}$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0} [2x+3] = 2(0)+3 = 3$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0} 3(x+1) = 3(0+1) = 3$$

$$\therefore \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0} f(x) = 3$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1} 3(x+1) = 3(1+1) = 6$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} 3(x+1) = 3(1+1) = 6$$

$$\therefore \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} f(x) = 6$$

## Question 24:

Find  $\lim_{x \rightarrow 1} f(x)$ , where  $f(x) = \begin{cases} x^2-1, & x \leq 1 \\ -x^2-1, & x > 1 \end{cases}$

## Answer 24:

The given function is



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$$f(x) = \begin{cases} x^2 - 1, & x \leq 1 \\ -x^2 - 1, & x > 1 \end{cases}$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1} [x^2 - 1] = 1^2 - 1 = 1 - 1 = 0$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} [-x^2 - 1] = -1^2 - 1 = -1 - 1 = -2$$

It is observed that  $\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$ .

Hence,  $\lim_{x \rightarrow 1} f(x)$  does not exist.

**Question 25:**

Evaluate  $\lim_{x \rightarrow 0} f(x)$ , where  $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

**Answer 25:**

The given function is  $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \left[ \frac{|x|}{x} \right]$$

$$= \lim_{x \rightarrow 0} \left( \frac{-x}{x} \right) \quad \left[ \text{When } x \text{ is negative, } |x| = -x \right]$$

$$= \lim_{x \rightarrow 0} (-1)$$

$$= -1$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \left[ \frac{|x|}{x} \right]$$

$$= \lim_{x \rightarrow 0} \left[ \frac{x}{x} \right] \quad \left[ \text{When } x \text{ is positive, } |x| = x \right]$$

$$= \lim_{x \rightarrow 0} (1)$$

$$= 1$$

It is observed that  $\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$ .

Hence,  $\lim_{x \rightarrow 0} f(x)$  does not exist.



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**Question 26:**

Find  $\lim_{x \rightarrow 0} f(x)$ , where  $f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

**Answer 26:**

The given function is

$$f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$\begin{aligned} \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} \left[ \frac{x}{|x|} \right] \\ &= \lim_{x \rightarrow 0^-} \left[ \frac{x}{-x} \right] && \text{[ When } x < 0, |x| = -x \text{]} \\ &= \lim_{x \rightarrow 0^-} (-1) \\ &= -1 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} \left[ \frac{x}{|x|} \right] \\ &= \lim_{x \rightarrow 0^+} \left[ \frac{x}{x} \right] && \text{[ When } x > 0, |x| = x \text{]} \\ &= \lim_{x \rightarrow 0^+} (1) \\ &= 1 \end{aligned}$$

It is observed that  $\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$ .

Hence,  $\lim_{x \rightarrow 0} f(x)$  does not exist.



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## Question 27:

Find  $\lim_{x \rightarrow 5} f(x)$ , where  $f(x) = |x| - 5$

## Answer 27:

The given function is  $f(x) = |x| - 5$ .

$$\begin{aligned}\lim_{x \rightarrow 5^-} f(x) &= \lim_{x \rightarrow 5^-} [|x| - 5] \\ &= \lim_{x \rightarrow 5^-} (x - 5) \quad [\text{When } x > 0, |x| = x] \\ &= 5 - 5 \\ &= 0\end{aligned}$$

$$\begin{aligned}\lim_{x \rightarrow 5^+} f(x) &= \lim_{x \rightarrow 5^+} (|x| - 5) \\ &= \lim_{x \rightarrow 5^+} (x - 5) \quad [\text{When } x > 0, |x| = x] \\ &= 5 - 5 \\ &= 0\end{aligned}$$

$$\therefore \lim_{x \rightarrow 5^-} f(x) = \lim_{x \rightarrow 5^+} f(x) = 0$$

$$\text{Hence, } \lim_{x \rightarrow 5} f(x) = 0$$



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## Question 28:

$$\text{Suppose } f(x) = \begin{cases} a + bx, & \text{if } x < 1 \\ 4, & \text{if } x = 0, \\ b - ax, & \text{if } x > 1 \end{cases}$$

and  $\lim_{x \rightarrow 1} f(x) = f(1)$  what are possible values of  $a$  and  $b$ ?

## Answer 28:

The given function is

$$f(x) = \begin{cases} a + bx, & x < 1 \\ 4, & x = 1 \\ b - ax & x > 1 \end{cases}$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1} (a + bx) = a + b$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} (b - ax) = b - a$$

$$f(1) = 4$$

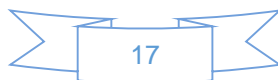
It is given that  $\lim_{x \rightarrow 1} f(x) = f(1)$ .

$$\therefore \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} f(x) = f(1)$$

$$\Rightarrow a + b = 4 \text{ and } b - a = 4$$

On solving these two equations, we obtain  $a = 0$  and  $b = 4$ .

Thus, the respective possible values of  $a$  and  $b$  are 0 and 4.



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## Question 29:

Let  $a_1, a_2, \dots, a_n$  be fixed real numbers and define a function

$$f(x) = (x - a_1)(x - a_2)\dots(x - a_n).$$

What is  $\lim_{x \rightarrow a_1} f(x)$ ? For some  $a \neq a_1, a_2, \dots, a_n$ , compute  $\lim_{x \rightarrow a} f(x)$ .

## Answer 29:

The given function is  $f(x) = (x - a_1)(x - a_2)\dots(x - a_n)$

$$\begin{aligned}\lim_{x \rightarrow a_1} f(x) &= \lim_{x \rightarrow a_1} [(x - a_1)(x - a_2)\dots(x - a_n)] \\ &= \left[ \lim_{x \rightarrow a_1} (x - a_1) \right] \left[ \lim_{x \rightarrow a_1} (x - a_2) \right] \dots \left[ \lim_{x \rightarrow a_1} (x - a_n) \right] \\ &= (a_1 - a_1)(a_1 - a_2)\dots(a_1 - a_n) = 0\end{aligned}$$

$$\therefore \lim_{x \rightarrow a_1} f(x) = 0$$

$$\begin{aligned}\text{Now, } \lim_{x \rightarrow a} f(x) &= \lim_{x \rightarrow a} [(x - a_1)(x - a_2)\dots(x - a_n)] \\ &= \left[ \lim_{x \rightarrow a} (x - a_1) \right] \left[ \lim_{x \rightarrow a} (x - a_2) \right] \dots \left[ \lim_{x \rightarrow a} (x - a_n) \right] \\ &= (a - a_1)(a - a_2)\dots(a - a_n)\end{aligned}$$

$$\therefore \lim_{x \rightarrow a} f(x) = (a - a_1)(a - a_2)\dots(a - a_n)$$

## Question 30:

$$\text{If } f(x) = \begin{cases} |x| + 1, & x < 0 \\ 0, & x = 0 \\ |x| - 1, & x > 0 \end{cases}$$

For what value(s) of  $a$  does  $\lim_{x \rightarrow a} f(x)$  exist?

## Answer 30:

The given function is



# Mathematics

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(Class – XI)

$$f(x) = \begin{cases} |x| + 1, & x < 0 \\ 0, & x = 0 \\ |x| - 1, & x > 0 \end{cases}$$

When  $a = 0$ ,

$$\begin{aligned} \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} (|x| + 1) \\ &= \lim_{x \rightarrow 0} (-x + 1) && [\text{If } x < 0, |x| = -x] \\ &= -0 + 1 \\ &= 1 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} (|x| - 1) \\ &= \lim_{x \rightarrow 0} (x - 1) && [\text{If } x > 0, |x| = x] \\ &= 0 - 1 \\ &= -1 \end{aligned}$$

Here, it is observed that  $\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$ .

$\therefore \lim_{x \rightarrow 0} f(x)$  does not exist.

When  $a < 0$ ,

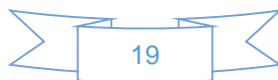
$$\begin{aligned} \lim_{x \rightarrow a^-} f(x) &= \lim_{x \rightarrow a^-} (|x| + 1) \\ &= \lim_{x \rightarrow a} (-x + 1) && [x < a < 0 \Rightarrow |x| = -x] \\ &= -a + 1 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow a^+} f(x) &= \lim_{x \rightarrow a^+} (|x| + 1) \\ &= \lim_{x \rightarrow a} (-x + 1) && [a < x < 0 \Rightarrow |x| = -x] \\ &= -a + 1 \end{aligned}$$

$$\therefore \lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = -a + 1$$

Thus, limit of  $f(x)$  exists at  $x = a$ , where  $a < 0$ .

When  $a > 0$



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$$\begin{aligned}\lim_{x \rightarrow a^-} f(x) &= \lim_{x \rightarrow a^-} (|x| - 1) \\ &= \lim_{x \rightarrow a^-} (x - 1) \quad [0 < x < a \Rightarrow |x| = x] \\ &= a - 1\end{aligned}$$

$$\begin{aligned}\lim_{x \rightarrow a^+} f(x) &= \lim_{x \rightarrow a^+} (|x| - 1) \\ &= \lim_{x \rightarrow a^+} (x - 1) \quad [0 < a < x \Rightarrow |x| = x] \\ &= a - 1\end{aligned}$$

$$\therefore \lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = a - 1$$

Thus, limit of  $f(x)$  exists at  $x = a$ , where  $a > 0$ .

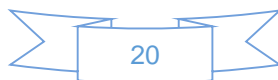
Thus,  $\lim_{x \rightarrow a} f(x)$  exists for all  $a \neq 0$ .

## Question 31:

If the function  $f(x)$  satisfies,  $\lim_{x \rightarrow 1} \frac{f(x) - 2}{x^2 - 1} = \pi$  evaluate  $\lim_{x \rightarrow 1} f(x)$ .

## Answer 31:

$$\begin{aligned}\lim_{x \rightarrow 1} \frac{f(x) - 2}{x^2 - 1} &= \pi \\ \Rightarrow \frac{\lim_{x \rightarrow 1} (f(x) - 2)}{\lim_{x \rightarrow 1} (x^2 - 1)} &= \pi \\ \Rightarrow \lim_{x \rightarrow 1} (f(x) - 2) &= \pi \lim_{x \rightarrow 1} (x^2 - 1) \\ \Rightarrow \lim_{x \rightarrow 1} (f(x) - 2) &= \pi(1^2 - 1) \\ \Rightarrow \lim_{x \rightarrow 1} (f(x) - 2) &= 0 \\ \Rightarrow \lim_{x \rightarrow 1} f(x) - \lim_{x \rightarrow 1} 2 &= 0 \\ \Rightarrow \lim_{x \rightarrow 1} f(x) - 2 &= 0 \\ \therefore \lim_{x \rightarrow 1} f(x) &= 2\end{aligned}$$



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## Question 32:

If,  $f(x) = \begin{cases} mx^2 + n, & x < 0 \\ nx + m, & 0 \leq x \leq 1 \\ nx^3 + m, & x > 1 \end{cases}$  For what integers  $m$  and  $n$  does

$\lim_{x \rightarrow 0} f(x)$  and  $\lim_{x \rightarrow 1} f(x)$  exist?

**Answer 32:** The given function is

$$f(x) = \begin{cases} mx^2 + n, & x < 0 \\ nx + m, & 0 \leq x \leq 1 \\ nx^3 + m, & x > 1 \end{cases}$$

$$\begin{aligned} \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} (mx^2 + n) \\ &= m(0)^2 + n \\ &= n \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} (nx + m) \\ &= n(0) + m \\ &= m. \end{aligned}$$

Thus,  $\lim_{x \rightarrow 0} f(x)$  exists if  $m = n$ .

$$\begin{aligned} \lim_{x \rightarrow 1^-} f(x) &= \lim_{x \rightarrow 1^-} (nx + m) \\ &= n(1) + m \\ &= m + n \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 1^+} f(x) &= \lim_{x \rightarrow 1^+} (nx^3 + m) \\ &= n(1)^3 + m \\ &= m + n \end{aligned}$$

$$\therefore \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} f(x).$$

Thus  $\lim_{x \rightarrow 1} f(x)$  exist for any integral value of  $m$  and  $n$ .