Mathematics

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(Chapter – 13) (Limits and Derivatives)

(Class – XI)

Exercise 13.2 (Supplementary)

Evaluate the following limits, if exist.

Question 1: $\lim_{x \to 0} \frac{e^{4x}-1}{x}$

Answer 1: $\lim_{x \to 0} \frac{e^{4x}-1}{x}$

$= \lim_{x \to 0} \frac{e^{4x}-1}{4x} \times 4$

$= \lim_{y \to 0} \frac{e^{y}-1}{y} \times 4$ $\quad [\,Where\ y = 4x\,]$

$= 1 \times 4$ $\quad \left[\text{Using } \lim_{y \to 0} \frac{e^{y}-1}{y} = 1\right]$

$= 4$

Question 2: $\lim_{x \to 0} \frac{e^{2+x}-e^{2}}{x}$

Answer 2: $\lim_{x \to 0} \frac{e^{2+x}-e^{2}}{x}$

$= \lim_{x \to 0} \frac{e^{2}(e^{x}-1)}{x}$

$= e^2 \times 1$ $\quad \left[\text{Using } \lim_{x \to 0} \frac{e^{x}-1}{x} = 1\right]$

$= e^2$

Question 3: $\lim_{x \to 5} \frac{e^{x}-e^{5}}{x-5}$

Answer 3: $\lim_{x \to 5} \frac{e^{x}-e^{5}}{x-5}$

$Put\ x = 5 + h, then\ as\ x \to 5 \Rightarrow h \longrightarrow 0. Therefore$

$\lim_{x \to 5} \frac{e^x - e^5}{x-5} = \lim_{h \to 0} \frac{e^{5+h} - e^5}{h}$

$= \lim_{h \to 0} \frac{e^5(e^h - 1)}{h}$

$= e^5 \times 1$ $\left[\text{Using } \lim_{h \to 0} \frac{e^h - 1}{h} = 1\right]$

$= e^5$

Question 4: $\lim_{x \to 0} \frac{e^{\sin x} - 1}{x}$

Answer 4: $\lim_{x \to 0} \frac{e^{\sin x} - 1}{x}$

$= \lim_{x \to 0} \frac{e^{\sin x} - 1}{x} \times \frac{\sin x}{\sin x}$

$= \lim_{x \to 0} \frac{e^{\sin x} - 1}{\sin x} \times \frac{\sin x}{x}$

$= \lim_{y \to 0} \frac{e^y - 1}{y} \times \lim_{x \to 0} \frac{\sin x}{x}$ $[Where\ y = \sin x]$

$= 1 \times 1$ $\left[\text{Using } \lim_{y \to 0} \frac{e^y - 1}{y} = 1\ and\ \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$

$= 1$

Question 5: $\lim_{x \to 3} \frac{e^x - e^3}{x - 3}$

Answer 5: $\lim_{x \to 3} \frac{e^x - e^3}{x-3}$

$Put\ x = 3 + h, then\ as\ x \to 3 \Rightarrow h \longrightarrow 0. Therefore$

$\lim_{x\to 3}\frac{e^x - 3}{x-3} = \lim_{h\to 0}\frac{e^{3+h}-e^3}{h}$

$= \lim_{h\to 0}\frac{e^3(e^h-1)}{h}$

$= e^3 \times 1$ $\quad\left[\text{Using } \lim_{h\to 0}\frac{e^h-1}{h} = 1\right]$

$= e^3$

Question 6: $\lim_{x\to 0}\frac{x\,(e^{\,x}-1)}{1-\cos x}$

Answer 6: $\lim_{x\to 0}\frac{x\,(e^{\,x}-1)}{1-\cos x}$

$= \lim_{x\to 0}\frac{x\,(e^{\,x}-1)}{1-\cos x}\times\frac{1+\cos x}{1+\cos x}\times\frac{x}{x}$

$= \lim_{x\to 0}\frac{(e^{\,x}-1)}{x}\times\frac{1+\cos x}{1}\times\frac{x^2}{1-cos^2x}$

$= \lim_{x\to 0}\frac{(e^{\,x}-1)}{x}\times\lim_{x\to 0}\frac{1+\cos x}{1}\times\lim_{x\to 0}\frac{x^2}{sin^2x}$

$= \lim_{x\to 0}\frac{(e^{\,x}-1)}{x}\times\lim_{x\to 0}\frac{1+\cos x}{1}\times\lim_{x\to 0}\frac{1}{\left(\frac{\sin x}{x}\right)^2}$

$= 1 \times (1+1)\times\frac{1}{1^1}$ $\quad\left[\text{Using } \lim_{x\to 0}\frac{e^x-1}{x} = 1 \text{ and } \lim_{x\to 0}\frac{\sin x}{x} = 1\right]$

$= 2$

Question 7: $\lim_{x\to 0}\frac{log_e\,(1+2x)}{x}$

Answer 7: $\lim_{x\to 0}\frac{log_e\,(1+2x)}{x}$

$= \lim_{x\to 0}\frac{log_e\,(1+2x)}{2x}\times 2$

$= \lim_{y \to 0} \frac{log_e(1+y)}{y} \times 2$ $\quad [Where \ \ y = 2x]$

$= 1 \times 2$ $\quad \left[Using \ \ \lim_{y \to 0} \frac{log_e(1+y)}{y} = 1\right]$

$= 2$

Question 8: $\lim_{x \to 0} \frac{log(1+x^3)}{sin^3 x}$

Answer 8: $\lim_{x \to 0} \frac{log(1+x^3)}{sin^3 x}$

$= \lim_{x \to 0} \frac{log(1+x^3)}{sin^3 x} \times \frac{x^3}{x^3}$

$= \lim_{x \to 0} \frac{log(1+x^3)}{x^3} \times \frac{x^3}{sin^3 x}$

$= \lim_{y \to 0} \frac{log(1+y)}{y} \times \lim_{x \to 0} \frac{1}{\left(\frac{\sin x}{x}\right)^3}$ $\quad [Where \ y = \ x^3]$

$= 1 \times \frac{1}{1^3}$ $\quad \left[Using \ \ \lim_{y \to 0} \frac{log(1+y)}{y} = 1 \ and \ \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$

$= 1$