

# Mathematics

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(Chapter – 13) (Limits and Derivatives)

(Class – XI)

## Exercise 13.2 (Supplementary)

Evaluate the following limits, if exist.

**Question 1:**  $\lim_{x \rightarrow 0} \frac{e^{4x} - 1}{x}$

**Answer 1:**  $\lim_{x \rightarrow 0} \frac{e^{4x} - 1}{x}$   
 $= \lim_{x \rightarrow 0} \frac{e^{4x} - 1}{4x} \times 4$

$= \lim_{y \rightarrow 0} \frac{e^y - 1}{y} \times 4$  [Where  $y = 4x$ ]

$= 1 \times 4$  [Using  $\lim_{y \rightarrow 0} \frac{e^y - 1}{y} = 1$ ]

$= 4$

**Question 2:**  $\lim_{x \rightarrow 0} \frac{e^{2+x} - e^2}{x}$

**Answer 2:**  $\lim_{x \rightarrow 0} \frac{e^{2+x} - e^2}{x}$   
 $= \lim_{x \rightarrow 0} \frac{e^2(e^x - 1)}{x}$

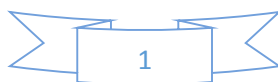
$= e^2 \times 1$  [Using  $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$ ]

$= e^2$

**Question 3:**  $\lim_{x \rightarrow 5} \frac{e^x - e^5}{x - 5}$

**Answer 3:**  $\lim_{x \rightarrow 5} \frac{e^x - e^5}{x - 5}$

Put  $x = 5 + h$ , then as  $x \rightarrow 5 \Rightarrow h \rightarrow 0$ . Therefore



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$$\begin{aligned}\lim_{x \rightarrow 5} \frac{e^x - e^5}{x - 5} &= \lim_{h \rightarrow 0} \frac{e^{5+h} - e^5}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^5(e^h - 1)}{h} \\ &= e^5 \times 1 \quad \left[ \text{Using } \lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1 \right] \\ &= e^5\end{aligned}$$

**Question 4:**  $\lim_{x \rightarrow 0} \frac{e^{\sin x} - 1}{x}$

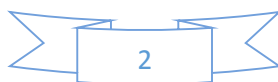
**Answer 4:**

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{e^{\sin x} - 1}{x} &= \lim_{x \rightarrow 0} \frac{e^{\sin x} - 1}{\sin x} \times \frac{\sin x}{x} \\ &= \lim_{x \rightarrow 0} \frac{e^{\sin x} - 1}{\sin x} \times \frac{\sin x}{x} \\ &= \lim_{y \rightarrow 0} \frac{e^y - 1}{y} \times \lim_{x \rightarrow 0} \frac{\sin x}{x} \quad [\text{Where } y = \sin x] \\ &= 1 \times 1 \quad \left[ \text{Using } \lim_{y \rightarrow 0} \frac{e^y - 1}{y} = 1 \text{ and } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \\ &= 1\end{aligned}$$

**Question 5:**  $\lim_{x \rightarrow 3} \frac{e^x - e^3}{x - 3}$

**Answer 5:**  $\lim_{x \rightarrow 3} \frac{e^x - e^3}{x - 3}$

Put  $x = 3 + h$ , then as  $x \rightarrow 3 \Rightarrow h \rightarrow 0$ . Therefore



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$$\begin{aligned}\lim_{x \rightarrow 3} \frac{e^x - 3}{x - 3} &= \lim_{h \rightarrow 0} \frac{e^{3+h} - e^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^3(e^h - 1)}{h} \\ &= e^3 \times 1 \quad \left[ \text{Using } \lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1 \right] \\ &= e^3\end{aligned}$$

**Question 6:**  $\lim_{x \rightarrow 0} \frac{x(e^x - 1)}{1 - \cos x}$

**Answer 6:**  $\lim_{x \rightarrow 0} \frac{x(e^x - 1)}{1 - \cos x}$

$$\begin{aligned}&= \lim_{x \rightarrow 0} \frac{x(e^x - 1)}{1 - \cos x} \times \frac{1 + \cos x}{1 + \cos x} \times \frac{x}{x} \\ &= \lim_{x \rightarrow 0} \frac{(e^x - 1)}{x} \times \frac{1 + \cos x}{1} \times \frac{x^2}{1 - \cos^2 x} \\ &= \lim_{x \rightarrow 0} \frac{(e^x - 1)}{x} \times \lim_{x \rightarrow 0} \frac{1 + \cos x}{1} \times \lim_{x \rightarrow 0} \frac{x^2}{\sin^2 x} \\ &= \lim_{x \rightarrow 0} \frac{(e^x - 1)}{x} \times \lim_{x \rightarrow 0} \frac{1 + \cos x}{1} \times \lim_{x \rightarrow 0} \frac{1}{\left(\frac{\sin x}{x}\right)^2} \\ &= 1 \times (1 + 1) \times \frac{1}{1^2} \quad \left[ \text{Using } \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1 \text{ and } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \\ &= 2\end{aligned}$$

**Question 7:**  $\lim_{x \rightarrow 0} \frac{\log_e(1+2x)}{x}$

**Answer 7:**  $\lim_{x \rightarrow 0} \frac{\log_e(1+2x)}{x}$

$$= \lim_{x \rightarrow 0} \frac{\log_e(1+2x)}{2x} \times 2$$

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$$= \lim_{y \rightarrow 0} \frac{\log_e (1+y)}{y} \times 2 \quad [\text{Where } y = 2x]$$

$$= 1 \times 2 \quad \left[ \text{Using } \lim_{y \rightarrow 0} \frac{\log_e (1+y)}{y} = 1 \right]$$

$$= 2$$

**Question 8:**  $\lim_{x \rightarrow 0} \frac{\log (1+x^3)}{\sin^3 x}$

**Answer 8:**  $\lim_{x \rightarrow 0} \frac{\log (1+x^3)}{\sin^3 x}$

$$= \lim_{x \rightarrow 0} \frac{\log (1+x^3)}{\sin^3 x} \times \frac{x^3}{x^3}$$

$$= \lim_{x \rightarrow 0} \frac{\log (1+x^3)}{x^3} \times \frac{x^3}{\sin^3 x}$$

$$= \lim_{y \rightarrow 0} \frac{\log (1+y)}{y} \times \lim_{x \rightarrow 0} \frac{1}{\left(\frac{\sin x}{x}\right)^3} \quad [\text{Where } y = x^3]$$

$$= 1 \times \frac{1}{1^3} \quad \left[ \text{Using } \lim_{y \rightarrow 0} \frac{\log (1+y)}{y} = 1 \text{ and } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

$$= 1$$