Mathematics

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(Chapter 10)(Vector Algebra)

XII

Miscellaneous Exercise 10

Question 1:

Write down a unit vector in XY-plane, making an angle of 30° with the positive direction of x-axis.

Answer 1:

If $\vec{r}$ is a unit vector in the XY-plane, then $\vec{r} = \cos\theta\hat{i} + \sin\theta\hat{j}.$

Here, θ is the angle made by the unit vector with the positive direction of the x-axis.

Therefore, for θ = 30°:

$$\vec{r} = \cos 30°\hat{i} + \sin 30°\hat{j} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$$

Hence, the required unit vector is $\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

Question 2:

Find the scalar components and magnitude of the vector joining the points $\mathrm{P}(x_1, y_1, z_1)$ and $\mathrm{Q}(x_2, y_2, z_2)$.

Answer 2:

The vector joining the points $\mathrm{P}(x_1, y_1, z_1)$ and $\mathrm{Q}(x_2, y_2, z_2)$

$$\overrightarrow{\mathrm{PQ}} = \text{Position vector of Q} - \text{Position vector of P}$$

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$\left|\overrightarrow{\mathrm{PQ}}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Hence, the scalar components and the magnitude of the vector joining the given points are respectively $\{(x_2 - x_1), (y_2 - y_1), (z_2 - z_1)\}$ and $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

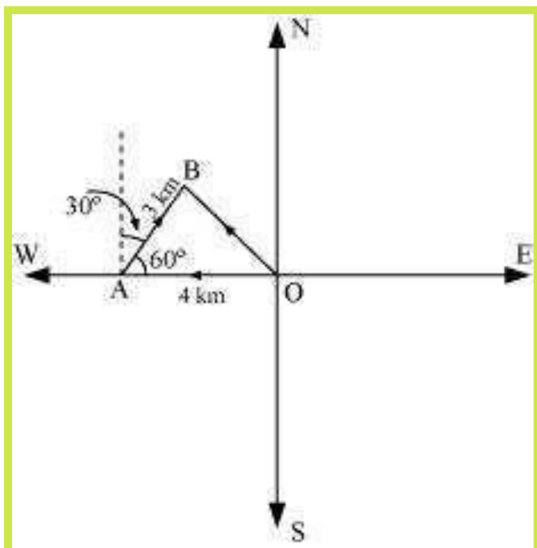
Question 3:

A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure.

Answer 3:

Let O and B be the initial and final positions of the girl respectively.

Then, the girl's position can be shown as:



Now, we have:

$$\overrightarrow{OA} = -4\hat{i}$$

$$\overrightarrow{AB} = \hat{i}\left|\overrightarrow{AB}\right|\cos 60° + \hat{j}\left|\overrightarrow{AB}\right|\sin 60°$$

$$= \hat{i}3 \times \frac{1}{2} + \hat{j}3 \times \frac{\sqrt{3}}{2}$$

$$= \frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

By the triangle law of vector addition, we have:

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$$
$$= \left(-4\hat{i}\right) + \left(\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}\right)$$
$$= \left(-4 + \frac{3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$
$$= \left(\frac{-8+3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$
$$= \frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

Hence, the girl's displacement from her initial point of departure is

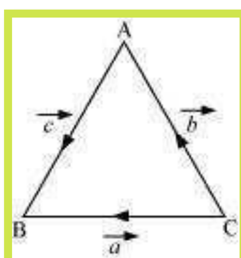
$\frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$.

Question 4:

If $\vec{a} = \vec{b} + \vec{c}$, then is it true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$? Justify your answer.

Answer 4:

In ΔABC, let $\overrightarrow{CB} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$, and $\overrightarrow{AB} = \vec{c}$ (as shown in the following figure).



Now, by the triangle law of vector addition, we have $\vec{a} = \vec{b} + \vec{c}$.

It is clearly known that $|\vec{a}|$, $|\vec{b}|$, and $|\vec{c}|$ represent the sides of ΔABC.

Also, it is known that the sum of the lengths of any two sides of a triangle is greater than the third side.

$$\therefore |\vec{a}| < |\vec{b}| + |\vec{c}|$$

Hence, it is not true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$.

Question 5:

Find the value of x for which $x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.

Answer 5:

$x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector if $|x(\hat{i} + \hat{j} + \hat{k})| = 1$.

Now,

$$|x(\hat{i} + \hat{j} + \hat{k})| = 1$$
$$\Rightarrow \sqrt{x^2 + x^2 + x^2} = 1$$
$$\Rightarrow \sqrt{3x^2} = 1$$
$$\Rightarrow \sqrt{3}\,x = 1$$
$$\Rightarrow x = \pm\frac{1}{\sqrt{3}}$$

Hence, the required value of x is $\pm\frac{1}{\sqrt{3}}$.

Question 6:

Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

Answer 6:

We have,

$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Let $\vec{c}$ be the resultant of $\vec{a}$ and $\vec{b}$.

Then,

$$\vec{c} = \vec{a} + \vec{b} = (2+1)\hat{i} + (3-2)\hat{j} + (-1+1)\hat{k} = 3\hat{i} + \hat{j}$$
$$\therefore |\vec{c}| = \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$$
$$\therefore \hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{(3\hat{i} + \hat{j})}{\sqrt{10}}$$

Hence, the vector of magnitude 5 units and parallel to the resultant of vectors $\vec{a}$ and $\vec{b}$ is

$$\pm 5\cdot\hat{c} = \pm 5\cdot\frac{1}{\sqrt{10}}\left(3\hat{i}+\hat{j}\right) = \pm\frac{3\sqrt{10}\hat{i}}{2} \pm \frac{\sqrt{10}}{2}\hat{j}.$$

Question 7:

If $\vec{a} = \hat{i}+\hat{j}+\hat{k},\ \vec{b} = 2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{c} = \hat{i}-2\hat{j}+\hat{k}$, find a unit vector parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$.

Answer 7:

We have,

$$\vec{a} = \hat{i}+\hat{j}+\hat{k},\ \vec{b} = 2\hat{i}-\hat{j}+3\hat{k} \text{ and } \vec{c} = \hat{i}-2\hat{j}+\hat{k}$$

$$\begin{aligned} 2\vec{a}-\vec{b}+3\vec{c} &= 2\left(\hat{i}+\hat{j}+\hat{k}\right)-\left(2\hat{i}-\hat{j}+3\hat{k}\right)+3\left(\hat{i}-2\hat{j}+\hat{k}\right) \\ &= 2\hat{i}+2\hat{j}+2\hat{k}-2\hat{i}+\hat{j}-3\hat{k}+3\hat{i}-6\hat{j}+3\hat{k} \\ &= 3\hat{i}-3\hat{j}+2\hat{k} \end{aligned}$$

$$\left|2\vec{a}-\vec{b}+3\vec{c}\right| = \sqrt{3^2+(-3)^2+2^2} = \sqrt{9+9+4} = \sqrt{22}$$

Hence, the unit vector along $2\vec{a}-\vec{b}+3\vec{c}$ is

$$\frac{2\vec{a}-\vec{b}+3\vec{c}}{\left|2\vec{a}-\vec{b}+3\vec{c}\right|} = \frac{3\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{22}} = \frac{3}{\sqrt{22}}\hat{i}-\frac{3}{\sqrt{22}}\hat{j}+\frac{2}{\sqrt{22}}\hat{k}.$$

Question 8:

Show that the points A (1, –2, –8), B (5, 0, –2) and C (11, 3, 7) are collinear, and find the ratio in which B divides AC.

Answer 8:

The given points are A (1, –2, –8), B (5, 0, –2), and C (11, 3, 7).

$$\therefore \overrightarrow{AB} = (5-1)\hat{i} + (0+2)\hat{j} + (-2+8)\hat{k} = 4\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\overrightarrow{BC} = (11-5)\hat{i} + (3-0)\hat{j} + (7+2)\hat{k} = 6\hat{i} + 3\hat{j} + 9\hat{k}$$

$$\overrightarrow{AC} = (11-1)\hat{i} + (3+2)\hat{j} + (7+8)\hat{k} = 10\hat{i} + 5\hat{j} + 15\hat{k}$$

$$\left|\overrightarrow{AB}\right| = \sqrt{4^2 + 2^2 + 6^2} = \sqrt{16+4+36} = \sqrt{56} = 2\sqrt{14}$$

$$\left|\overrightarrow{BC}\right| = \sqrt{6^2 + 3^2 + 9^2} = \sqrt{36+9+81} = \sqrt{126} = 3\sqrt{14}$$

$$\left|\overrightarrow{AC}\right| = \sqrt{10^2 + 5^2 + 15^2} = \sqrt{100+25+225} = \sqrt{350} = 5\sqrt{14}$$

$$\therefore \left|\overrightarrow{AC}\right| = \left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right|$$

Thus, the given points A, B, and C are collinear.

Now, let point B divide AC in the ratio $\lambda : 1$ Then, we have:

$$\overrightarrow{OB} = \frac{\lambda\overrightarrow{OC} + \overrightarrow{OA}}{(\lambda+1)}$$

$$\Rightarrow 5\hat{i} - 2\hat{k} = \frac{\lambda\left(11\hat{i} + 3\hat{j} + 7\hat{k}\right) + \left(\hat{i} - 2\hat{j} - 8\hat{k}\right)}{\lambda+1}$$

$$\Rightarrow (\lambda+1)\left(5\hat{i} - 2\hat{k}\right) = 11\lambda\hat{i} + 3\lambda\hat{j} + 7\lambda\hat{k} + \hat{i} - 2\hat{j} - 8\hat{k}$$

$$\Rightarrow 5(\lambda+1)\hat{i} - 2(\lambda+1)\hat{k} = (11\lambda+1)\hat{i} + (3\lambda-2)\hat{j} + (7\lambda-8)\hat{k}$$

On equating the corresponding components, we get:

$$5(\lambda+1) = 11\lambda + 1$$

$$\Rightarrow 5\lambda + 5 = 11\lambda + 1$$

$$\Rightarrow 6\lambda = 4$$

$$\Rightarrow \lambda = \frac{4}{6} = \frac{2}{3}$$

Hence, point B divides AC in the ratio $2:3.$

Question 9:

Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\left(2\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - 3\vec{b}\right)$ externally in the ratio 1: 2. Also, show that P is the mid point of the line segment RQ.

Answer 9:

It is given that $\overrightarrow{OP} = 2\vec{a} + \vec{b},\ \overrightarrow{OQ} = \vec{a} - 3\vec{b}$.

It is given that point R divides a line segment joining two points P and Q externally in the ratio 1: 2. Then, on using the section formula, we get:

$$\overrightarrow{OR} = \frac{2\left(2\vec{a} + \vec{b}\right) - \left(\vec{a} - 3\vec{b}\right)}{2-1} = \frac{4\vec{a} + 2\vec{b} - \vec{a} + 3\vec{b}}{1} = 3\vec{a} + 5\vec{b}$$

Therefore, the position vector of point R is $3\vec{a} + 5\vec{b}$

Position vector of the mid-point of RQ = $\frac{\overrightarrow{OQ} + \overrightarrow{OR}}{2}$

$$= \frac{\left(\vec{a} - 3\vec{b}\right) + \left(3\vec{a} + 5\vec{b}\right)}{2}$$

$$= 2\vec{a} + \vec{b}$$

$$= \overrightarrow{OP}$$

Hence, P is the mid-point of the line segment RQ.

Question 10:

The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$.

Find the unit vector parallel to its diagonal. Also, find its area.

Answer 10:

Adjacent sides of a parallelogram are given as: $\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - 3\hat{k}$

Then, the diagonal of a parallelogram is given by $\vec{a} + \vec{b}$

$$\vec{a} + \vec{b} = (2+1)\hat{i} + (-4-2)\hat{j} + (5-3)\hat{k} = 3\hat{i} - 6\hat{j} + 2\hat{k}$$

Thus, the unit vector parallel to the diagonal is

$$\frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|}=\frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{3^2+(-6)^2+2^2}}=\frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{9+36+4}}=\frac{3\hat{i}-6\hat{j}+2\hat{k}}{7}=\frac{3}{7}\hat{i}-\frac{6}{7}\hat{j}+\frac{2}{7}\hat{k}.$$

$\therefore$ Area of parallelogram ABCD = $|\vec{a}\times\vec{b}|$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -4 & 5\\ 1 & -2 & -3\end{vmatrix}$$

$$=\hat{i}(12+10)-\hat{j}(-6-5)+\hat{k}(-4+4)$$

$$=22\hat{i}+11\hat{j}$$

$$=11(2\hat{i}+\hat{j})$$

$$\therefore |\vec{a}\times\vec{b}|=11\sqrt{2^2+1^2}=11\sqrt{5}$$

Hence, the area of the parallelogram is $11\sqrt{5}$ square units.

Question 11:

Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ are $\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$.

Answer 11:

Let a vector be equally inclined to axes OX, OY, and OZ at angle α.

Then, the direction cosines of the vector are cos α, cos α, and cos α.

Now,

$$\cos^2\alpha+\cos^2\alpha+\cos^2\alpha=1$$

$$\Rightarrow 3\cos^2\alpha=1$$

$$\Rightarrow \cos\alpha=\frac{1}{\sqrt{3}}$$

Hence, the direction cosines of the vector which are equally inclined to the axes are $\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$.

Question 12:

Let $\vec{a}=\hat{i}+4\hat{j}+2\hat{k},\ \vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}.\vec{d}=15$.

Answer 12:

Let $\vec{d}=d_1\hat{i}+d_2\hat{j}+d_3\hat{k}$

Since $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$

$\vec{d}\cdot\vec{a}=0$

$\Rightarrow d_1+4d_2+2d_3=0 \qquad ...(i)$

And,

$\vec{d}\cdot\vec{b}=0$

$\Rightarrow 3d_1-2d_2+7d_3=0 \qquad ...(ii)$

Also, it is given that:

$\vec{c}\cdot\vec{d}=15$

$\Rightarrow 2d_1-d_2+4d_3=15 \qquad ...(iii)$

On solving (i), (ii), and (iii), we get:

$d_1=\frac{160}{3}, d_2=-\frac{5}{3}$ and $d_3=-\frac{70}{3}$

$\therefore \vec{d}=\frac{160}{3}\hat{i}-\frac{5}{3}\hat{j}-\frac{70}{3}\hat{k}=\frac{1}{3}\left(160\hat{i}-5\hat{j}-70\hat{k}\right)$

Hence, the required vector is $\frac{1}{3}\left(160\hat{i}-5\hat{j}-70\hat{k}\right)$

Question 13:

The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ.

Answer 13:

$\left(2\hat{i}+4\hat{j}-5\hat{k}\right)+\left(\lambda\hat{i}+2\hat{j}+3\hat{k}\right)$

$=(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}$

Therefore, unit vector along $\left(2\hat{i}+4\hat{j}-5\hat{k}\right)+\left(\lambda\hat{i}+2\hat{j}+3\hat{k}\right)$ is given as:

$$\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+6^2+(-2)^2}}=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{4+4\lambda+\lambda^2+36+4}}=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}$$

Scalar product of $\left(\hat{i}+\hat{j}+\hat{k}\right)$ with this unit vector is 1.

$$\Rightarrow\left(\hat{i}+\hat{j}+\hat{k}\right)\cdot\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}=1$$

$$\Rightarrow\frac{(2+\lambda)+6-2}{\sqrt{\lambda^2+4\lambda+44}}=1$$

$$\Rightarrow\sqrt{\lambda^2+4\lambda+44}=\lambda+6$$

$$\Rightarrow\lambda^2+4\lambda+44=(\lambda+6)^2$$

$$\Rightarrow\lambda^2+4\lambda+44=\lambda^2+12\lambda+36$$

$$\Rightarrow 8\lambda=8$$

$$\Rightarrow\lambda=1$$

Hence, the value of λ is 1.

Question 14:

If $\vec{a},\vec{b},\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a},\vec{b}$ and $\vec{c}$.

Answer 14:

Since $\vec{a},\vec{b},$ and $\vec{c}$ are mutually perpendicular vectors, we have $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0.$ It is given that: $|\vec{a}|=|\vec{b}|=|\vec{c}|$

Let vector $\vec{a}+\vec{b}+\vec{c}$ be inclined to $\vec{a},\vec{b},$ and $\vec{c}$ at angles $\theta_1,\ \theta_2,$ and θ_3 respectively.

Then, we have:

$$\cos\theta_1 = \frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|} = \frac{\vec{a}\cdot\vec{a}+\vec{b}\cdot\vec{a}+\vec{c}\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}$$

$$= \frac{|\vec{a}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|} \qquad [\vec{b}\cdot\vec{a}=\vec{c}\cdot\vec{a}=0]$$

$$= \frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

$$\cos\theta_2 = \frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{b}}{|\vec{a}+\vec{b}+\vec{c}||\vec{b}|} = \frac{\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b}+\vec{c}\cdot\vec{b}}{|\vec{a}+\vec{b}+\vec{c}|\cdot|\vec{b}|}$$

$$= \frac{|\vec{b}|^2}{|\vec{a}+\vec{b}+\vec{c}|\cdot|\vec{b}|} \qquad [\vec{a}\cdot\vec{b}=\vec{c}\cdot\vec{b}=0]$$

$$= \frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

$$\cos\theta_3 = \frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{c}}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|} = \frac{\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{c}}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|}$$

$$= \frac{|\vec{c}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|} \qquad [\vec{a}\cdot\vec{c}=\vec{b}\cdot\vec{c}=0]$$

$$= \frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

Now, as $|\vec{a}|=|\vec{b}|=|\vec{c}|$, $\cos\theta_1=\cos\theta_2=\cos\theta_3$

$\therefore \theta_1=\theta_2=\theta_3$

Hence, the vector $(\vec{a}+\vec{b}+\vec{c})$ is equally inclined to $\vec{a}, \vec{b}$, and $\vec{c}$

Question 15:

Prove that , $(\vec{a}+\vec{b}).(\vec{a}+\vec{b})=|\vec{a}|^2+|\vec{b}|^2$ if and only if $\vec{a}, \vec{b}$ are perpendicular, given $\vec{a}\neq\vec{0},\ \vec{b}\neq\vec{0}$.

Answer 15:

$(\vec{a}+\vec{b}).(\vec{a}+\vec{b})=|\vec{a}|^2+|\vec{b}|^2$

$\Leftrightarrow \vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b}=|\vec{a}|^2+|\vec{b}|^2$ [Distributivity of scalar products over addition]

$\Leftrightarrow |\vec{a}|^2+2\vec{a}\cdot\vec{b}+|\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2$ $[\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$ (Scalar product is commutative)$]$

$\Leftrightarrow 2\vec{a}\cdot\vec{b}=0$

$\Leftrightarrow \vec{a}\cdot\vec{b}=0$

$\therefore \vec{a}$ and $\vec{b}$ are perpendicular. $[\vec{a}\neq\vec{0},\ \vec{b}\neq\vec{0}$ (Given)$]$

Question 16:

If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a}.\vec{b}\geq 0$ only when

(A) $0<\theta<\frac{\pi}{2}$ (B) $0\leq\theta\leq\frac{\pi}{2}$

(C) $0<\theta<\pi$ (D) $0\leq\theta\leq\pi$

Answer 16:

Let θ be the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then, without loss of generality, $\vec{a}$ and $\vec{b}$ are non-zero vectors so that $|\vec{a}|$ and $|\vec{b}|$ are positive.

It is known that $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

$\therefore \vec{a}\cdot\vec{b}\geq 0$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta \geq 0$

$\Rightarrow \cos\theta \geq 0 \qquad \left[|\vec{a}| \text{ and } |\vec{b}| \text{ are positive}\right]$

$\Rightarrow 0 \leq \theta \leq \frac{\pi}{2}$

Hence, $\vec{a}.\vec{b}\geq 0$ when $0\leq\theta\leq\frac{\pi}{2}$

The correct answer is B.

Question 17:

Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a}+\vec{b}$ is a unit vector if

(A) $\theta=\frac{\pi}{4}$ (B) $\theta=\frac{\pi}{3}$ (C) $\theta=\frac{\pi}{2}$ (D) $\theta=\frac{2\pi}{3}$

Answer 17:

Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ be the angle between them.

Then, $|\vec{a}|=|\vec{b}|=1$.

Now, $\vec{a}+\vec{b}$ is a unit vector if $|\vec{a}+\vec{b}|=1$.

$\left|\vec{a}+\vec{b}\right|=1$

$\Rightarrow\left(\vec{a}+\vec{b}\right)^2=1$

$\Rightarrow\left(\vec{a}+\vec{b}\right).\left(\vec{a}+\vec{b}\right)=1$

$\Rightarrow\vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{b}.\vec{a}+\vec{b}.\vec{b}=1$

$\Rightarrow\left|\vec{a}\right|^2+2\vec{a}.\vec{b}+\left|\vec{b}\right|^2=1$

$\Rightarrow 1^2+2\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta+1^2=1$

$\Rightarrow 1+2.1.1\cos\theta+1=1$

$\Rightarrow\cos\theta=-\frac{1}{2}$

$\Rightarrow\theta=\frac{2\pi}{3}$

Hence, $\vec{a}+\vec{b}$ is a unit vector if $\theta=\frac{2\pi}{3}$

.

The correct answer is D.

Question 18:

The value of $\hat{i}.\left(\hat{j}\times\hat{k}\right)+\hat{j}.\left(\hat{i}\times\hat{k}\right)+\hat{k}.\left(\hat{i}\times\hat{j}\right)$ is

(A) 0 (B) −1 (C) 1 (D) 3

Answer 18:

$\hat{i}.\left(\hat{j}\times\hat{k}\right)+\hat{j}.\left(\hat{i}\times\hat{k}\right)+\hat{k}.\left(\hat{i}\times\hat{j}\right)$

$=\hat{i}\cdot\hat{i}+\hat{j}\cdot\left(-\hat{j}\right)+\hat{k}\cdot\hat{k}$

$=1-\hat{j}\cdot\hat{j}+1$

$=1-1+1$

$=1$

The correct answer is C.

Question 19:

If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$ when θ is equal to

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

Answer 19:

Let θ be the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then, without loss of generality, $\vec{a}$ and $\vec{b}$ are non-zero vectors, so that $|\vec{a}|$ and $|\vec{b}|$ are positive.

$$|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$$

$$\Rightarrow |\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\sin\theta$$

$$\Rightarrow \cos\theta = \sin\theta \qquad \left[|\vec{a}| \text{ and } |\vec{b}| \text{ are positive}\right]$$

$$\Rightarrow \tan\theta = 1$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Hence, $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$ when θ Is equal to $\frac{\pi}{4}$.

The correct answer is B.