

Mathematics

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(Chapter 11)(Three Dimensional Geometry)

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Miscellaneous Solutions

Question 1:

Show that the line joining the origin to the point $(2, 1, 1)$ is perpendicular to the line determined by the points $(3, 5, -1)$, $(4, 3, -1)$.

Answer

Let OA be the line joining the origin, O $(0, 0, 0)$, and the point, A $(2, 1, 1)$.

Also, let BC be the line joining the points, B $(3, 5, -1)$ and C $(4, 3, -1)$.

The direction ratios of OA are 2, 1, and 1 and of BC are $(4 - 3) = 1$, $(3 - 5) = -2$, and $(-1 + 1) = 0$

OA is perpendicular to BC, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$\therefore a_1a_2 + b_1b_2 + c_1c_2 = 2 \times 1 + 1(-2) + 1 \times 0 = 2 - 2 = 0$$

Thus, OA is perpendicular to BC.

Question 2:

If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both of these are $m_1n_2 - m_2n_1$, $n_1l_2 - n_2l_1$, $l_1m_2 - l_2m_1$.

Answer

It is given that l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines. Therefore,



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$$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0 \quad \dots(1)$$

$$l_1^2 + m_1^2 + n_1^2 = 1 \quad \dots(2)$$

$$l_2^2 + m_2^2 + n_2^2 = 1 \quad \dots(3)$$

Let l, m, n be the direction cosines of the line which is perpendicular to the line with direction cosines l_1, m_1, n_1 and l_2, m_2, n_2 .

$$\therefore ll_1 + mm_1 + nn_1 = 0$$

$$ll_2 + mm_2 + nn_2 = 0$$

$$\therefore \frac{l}{m_1 n_2 - m_2 n_1} = \frac{m}{n_1 l_2 - n_2 l_1} = \frac{n}{l_1 m_2 - l_2 m_1}$$

$$\Rightarrow \frac{l^2}{(m_1 n_2 - m_2 n_1)^2} = \frac{m^2}{(n_1 l_2 - n_2 l_1)^2} = \frac{n^2}{(l_1 m_2 - l_2 m_1)^2}$$

$$\Rightarrow \frac{l^2}{(m_1 n_2 - m_2 n_1)^2} = \frac{m^2}{(n_1 l_2 - n_2 l_1)^2} = \frac{n^2}{(l_1 m_2 - l_2 m_1)^2}$$

$$= \frac{l^2 + m^2 + n^2}{(m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - n_2 l_1)^2 + (l_1 m_2 - l_2 m_1)^2} \quad \dots(4)$$

l, m, n are the direction cosines of the line. $\therefore l^2 + m^2 + n^2 = 1 \dots (5)$

It is known that,

$$\begin{aligned} (l_1^2 + m_1^2 + n_1^2)(l_2^2 + m_2^2 + n_2^2) - (l_1 l_2 + m_1 m_2 + n_1 n_2)^2 \\ = (m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - n_2 l_1)^2 + (l_1 m_2 - l_2 m_1)^2 \end{aligned}$$

From (1), (2), and (3), we obtain

$$\Rightarrow 1 \cdot 1 - 0 = (m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - n_2 l_1)^2 + (l_1 m_2 - l_2 m_1)^2$$

$$\therefore (m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - n_2 l_1)^2 + (l_1 m_2 - l_2 m_1)^2 = 1 \quad \dots(6)$$

Substituting the values from equations (5) and (6) in equation (4), we obtain

$$\frac{l^2}{(m_1 n_2 - m_2 n_1)^2} = \frac{m^2}{(n_1 l_2 - n_2 l_1)^2} = \frac{n^2}{(l_1 m_2 - l_2 m_1)^2} = 1$$

$$\Rightarrow l = m_1 n_2 - m_2 n_1, m = n_1 l_2 - n_2 l_1, n = l_1 m_2 - l_2 m_1$$

Thus, the direction cosines of the required line are $m_1 n_2 - m_2 n_1, n_1 l_2 - n_2 l_1$, and $l_1 m_2 - l_2 m_1$.



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Question 3:

Find the angle between the lines whose direction ratios are a, b, c and $b - c, c - a, a - b$.

Answer

The angle Q between the lines with direction cosines, a, b, c and $b - c, c - a, a - b$, is given by,

$$\cos Q = \frac{a(b-c) + b(c-a) + c(a-b)}{\sqrt{a^2 + b^2 + c^2} \sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}}$$

$$\Rightarrow \cos Q = 0$$

$$\Rightarrow Q = \cos^{-1} 0$$

$$\Rightarrow Q = 90^\circ$$

Thus, the angle between the lines is 90° .

Question 4:

Find the equation of a line parallel to x-axis and passing through the origin.

Answer

The line parallel to x-axis and passing through the origin is x-axis itself.

Let A be a point on x-axis. Therefore, the coordinates of A are given by $(a, 0, 0)$, where $a \in \mathbb{R}$.

Direction ratios of OA are $(a - 0) = a, 0, 0$

The equation of OA is given by,

$$\frac{x-0}{a} = \frac{y-0}{0} = \frac{z-0}{0}$$

$$\Rightarrow \frac{x}{1} = \frac{y}{0} = \frac{z}{0} = a$$

Thus, the equation of line parallel to x-axis and passing through origin is

$$\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$$



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Question 5:

If the coordinates of the points A, B, C, D be (1, 2, 3), (4, 5, 7), (-4, 3, -6) and (2, 9, 2) respectively, then find the angle between the lines AB and CD.

Answer

The coordinates of A, B, C, and D are (1, 2, 3), (4, 5, 7), (-4, 3, -6), and (2, 9, 2) respectively.

The direction ratios of AB are $(4 - 1) = 3$, $(5 - 2) = 3$, and $(7 - 3) = 4$

The direction ratios of CD are $(2 - (-4)) = 6$, $(9 - 3) = 6$, and $(2 - (-6)) = 8$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$$

It can be seen that,

Therefore, AB is parallel to CD.

Thus, the angle between AB and CD is either 0° or 180° .

Question 6:

$$\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2} \quad \text{and} \quad \frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$$

If the lines are perpendicular, find the value of k.

Answer

$$\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2} \quad \text{and} \quad \frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$$

The direction ratios of the lines, $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$, are -3, 2k, 2 and 3k, 1, -5 respectively.

It is known that two lines with direction ratios, a_1, b_1, c_1 and a_2, b_2, c_2 , are perpendicular, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$\therefore -3(3k) + 2k \times 1 + 2(-5) = 0$$

$$\Rightarrow -9k + 2k - 10 = 0$$

$$\Rightarrow 7k = -10$$

$$\Rightarrow k = \frac{-10}{7}$$

Therefore, for $k = \frac{-10}{7}$, the given lines are perpendicular to each other.



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Question 7:

Find the vector equation of the plane passing through (1, 2, 3) and perpendicular to the

plane $\vec{r} \cdot (\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$

Answer

The position vector of the point (1, 2, 3) is $\vec{r}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$

The direction ratios of the normal to the $\vec{r} \cdot (\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$ plane, are 1, 2, and -5

and the normal vector is $\vec{N} = \hat{i} + 2\hat{j} - 5\hat{k}$

The equation of a line passing through a point and perpendicular to the given plane is

given by, $\vec{l} = \vec{r} + \lambda \vec{N}, \lambda \in R$

$$\Rightarrow \vec{l} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 5\hat{k})$$

Question 8:

Find the equation of the plane passing through (a, b, c) and parallel to the plane

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$$

Answer

Any plane parallel to the plane, $\vec{r}_1 \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = \lambda \quad \dots(1)$$

The plane passes through the point (a, b, c). Therefore, the position vector \vec{r} of this

point is $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$

Therefore, equation (1) becomes

$$(a\hat{i} + b\hat{j} + c\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = \lambda$$

$$\Rightarrow a + b + c = \lambda$$

Substituting $\lambda = a + b + c$ in equation (1), we obtain

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = a + b + c \quad \dots(2)$$



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This is the vector equation of the required plane.

Substituting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in equation (2), we obtain

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = a + b + c$$

$$\Rightarrow x + y + z = a + b + c$$

Question 9:

Find the shortest distance between lines $\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$

and $\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$.

Answer

The given lines are

$$\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k}) \quad \dots(1)$$

$$\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k}) \quad \dots(2)$$



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It is known that the shortest distance between two lines, $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda \vec{b}_2$, is given by

$$d = \frac{|(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}_1 \times \vec{b}_2|} \quad \dots(3)$$

Comparing $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda \vec{b}_2$ to equations (1) and (2), we obtain

$$\vec{a}_1 = 6\hat{i} + 2\hat{j} + 2\hat{k}$$

$$\vec{b}_1 = \hat{i} - 2\hat{j} + 2\hat{k}$$

$$\vec{a}_2 = -4\hat{i} - \hat{k}$$

$$\vec{b}_2 = 3\hat{i} - 2\hat{j} - 2\hat{k}$$

$$\Rightarrow \vec{a}_2 - \vec{a}_1 = (-4\hat{i} - \hat{k}) - (6\hat{i} + 2\hat{j} + 2\hat{k}) = -10\hat{i} - 2\hat{j} - 3\hat{k}$$

$$\Rightarrow \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix} = (4+4)\hat{i} - (-2-6)\hat{j} + (-2+6)\hat{k} = 8\hat{i} + 8\hat{j} + 4\hat{k}$$

$$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{(8)^2 + (8)^2 + (4)^2} = 12$$

$$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = (8\hat{i} + 8\hat{j} + 4\hat{k}) \cdot (-10\hat{i} - 2\hat{j} - 3\hat{k}) = -80 - 16 - 12 = -108$$

Substituting all the values in equation (1), we obtain

$$d = \frac{|-108|}{12} = 9$$

Therefore, the shortest distance between the two given lines is 9 units.

Question 10:

Find the coordinates of the point where the line through (5, 1, 6) and

(3, 4, 1) crosses the YZ-plane

Answer

It is known that the equation of the line passing through the points, (x_1, y_1, z_1) and $(x_2,$



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$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$
 (y_2, z_2) , is

The line passing through the points, (5, 1, 6) and (3, 4, 1), is given by,

$$\frac{x-5}{3-5} = \frac{y-1}{4-1} = \frac{z-6}{1-6}$$

$$\Rightarrow \frac{x-5}{-2} = \frac{y-1}{3} = \frac{z-6}{-5} = k \text{ (say)}$$

$$\Rightarrow x = 5 - 2k, y = 3k + 1, z = 6 - 5k$$

Any point on the line is of the form (5 - 2k, 3k + 1, 6 - 5k).

The equation of YZ-plane is $x = 0$

Since the line passes through YZ-plane,

$$5 - 2k = 0$$

$$\Rightarrow k = \frac{5}{2}$$

$$\Rightarrow 3k + 1 = 3 \times \frac{5}{2} + 1 = \frac{17}{2}$$

$$6 - 5k = 6 - 5 \times \frac{5}{2} = \frac{-13}{2}$$

Therefore, the required point is $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$.

Question 11:

Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the ZX - plane.

Answer

It is known that the equation of the line passing through the points, (x_1, y_1, z_1) and $(x_2,$

$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$
 $y_2, z_2)$, is

The line passing through the points, (5, 1, 6) and (3, 4, 1), is given by,



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$$\frac{x-5}{3-5} = \frac{y-1}{4-1} = \frac{z-6}{1-6}$$

$$\Rightarrow \frac{x-5}{-2} = \frac{y-1}{3} = \frac{z-6}{-5} = k \text{ (say)}$$

$$\Rightarrow x = 5 - 2k, y = 3k + 1, z = 6 - 5k$$

Any point on the line is of the form $(5 - 2k, 3k + 1, 6 - 5k)$.

Since the line passes through ZX-plane,

$$3k + 1 = 0$$

$$\Rightarrow k = -\frac{1}{3}$$

$$\Rightarrow 5 - 2k = 5 - 2\left(-\frac{1}{3}\right) = \frac{17}{3}$$

$$6 - 5k = 6 - 5\left(-\frac{1}{3}\right) = \frac{23}{3}$$

Therefore, the required point is $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$

Question 12:

Find the coordinates of the point where the line through $(3, -4, -5)$ and $(2, -3, 1)$ crosses the plane $2x + y + z = 7$.

Answer

It is known that the equation of the line through the points, (x_1, y_1, z_1) and (x_2, y_2, z_2) , is

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

Since the line passes through the points, $(3, -4, -5)$ and $(2, -3, 1)$, its equation is given by,

$$\frac{x-3}{2-3} = \frac{y+4}{-3+4} = \frac{z+5}{1+5}$$

$$\Rightarrow \frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = k \text{ (say)}$$

$$\Rightarrow x = 3 - k, y = k - 4, z = 6k - 5$$



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Therefore, any point on the line is of the form $(3 - k, k - 4, 6k - 5)$.

This point lies on the plane, $2x + y + z = 7$

$$\therefore 2(3 - k) + (k - 4) + (6k - 5) = 7$$

$$\Rightarrow 5k - 3 = 7$$

$$\Rightarrow k = 2$$

Hence, the coordinates of the required point are $(3 - 2, 2 - 4, 6 \times 2 - 5)$ i.e., $(1, -2, 7)$.

Question 13:

Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

Answer

The equation of the plane passing through the point $(-1, 3, 2)$ is $a(x + 1) + b(y - 3) + c(z - 2) = 0$... (1) where, a, b, c are the direction ratios of normal to the plane.

It is known that two planes, $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$, are perpendicular, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Plane (1) is perpendicular to the plane, $x + 2y + 3z = 5$

$$\therefore a \cdot 1 + b \cdot 2 + c \cdot 3 = 0$$

$$\Rightarrow a + 2b + 3c = 0 \quad \dots(2)$$

Also, plane (1) is perpendicular to the plane, $3x + 3y + z = 0$

$$\therefore a \cdot 3 + b \cdot 3 + c \cdot 1 = 0$$

$$\Rightarrow 3a + 3b + c = 0 \quad \dots(3)$$

From equations (2) and (3), we obtain

$$\frac{a}{2 \times 1 - 3 \times 3} = \frac{b}{3 \times 3 - 1 \times 1} = \frac{c}{1 \times 3 - 2 \times 3}$$

$$\Rightarrow \frac{a}{-7} = \frac{b}{8} = \frac{c}{-3} = k \text{ (say)}$$

$$\Rightarrow a = -7k, b = 8k, c = -3k$$

Substituting the values of $a, b,$ and c in equation (1), we obtain



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$$\begin{aligned} -7k(x+1)+8k(y-3)-3k(z-2) &= 0 \\ \Rightarrow (-7x-7)+(8y-24)-3z+6 &= 0 \\ \Rightarrow -7x+8y-3z-25 &= 0 \\ \Rightarrow 7x-8y+3z+25 &= 0 \end{aligned}$$

This is the required equation of the plane.

Question 14:

If the points $(1, 1, p)$ and $(-3, 0, 1)$ be equidistant from the plane

$$\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0, \text{ then find the value of } p.$$

Answer

The position vector through the point $(1, 1, p)$ is $\vec{a}_1 = \hat{i} + \hat{j} + p\hat{k}$

Similarly, the position vector through the point $(-3, 0, 1)$ is

$$\vec{a}_2 = -4\hat{i} + \hat{k}$$

The equation of the given plane is $\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$

It is known that the perpendicular distance between a point whose position vector is

$$\vec{a} \text{ and the plane, } \vec{r} \cdot \vec{N} = d, \text{ is given by, } D = \frac{|\vec{a} \cdot \vec{N} - d|}{|\vec{N}|}$$

Here, $\vec{N} = 3\hat{i} + 4\hat{j} - 12\hat{k}$ and $d = -13$

Therefore, the distance between the point $(1, 1, p)$ and the given plane is

$$\begin{aligned} D_1 &= \frac{|(\hat{i} + \hat{j} + p\hat{k}) \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13|}{|3\hat{i} + 4\hat{j} - 12\hat{k}|} \\ \Rightarrow D_1 &= \frac{|3+4-12p+13|}{\sqrt{3^2+4^2+(-12)^2}} \\ \Rightarrow D_1 &= \frac{|20-12p|}{13} \quad \dots(1) \end{aligned}$$



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Similarly, the distance between the point $(-3, 0, 1)$ and the given plane is

$$D_2 = \frac{|(-3\hat{i} + \hat{k}) \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13|}{|3\hat{i} + 4\hat{j} - 12\hat{k}|}$$
$$\Rightarrow D_2 = \frac{|-9 - 12 + 13|}{\sqrt{3^2 + 4^2 + (-12)^2}}$$
$$\Rightarrow D_2 = \frac{8}{13} \quad \dots(2)$$

It is given that the distance between the required plane and the points, $(1, 1, p)$ and $(-3, 0, 1)$, is equal.

$$\therefore D_1 = D_2$$

$$\Rightarrow \frac{|20 - 12p|}{13} = \frac{8}{13}$$
$$\Rightarrow 20 - 12p = 8 \text{ or } -(20 - 12p) = 8$$
$$\Rightarrow 12p = 12 \text{ or } 12p = 28$$
$$\Rightarrow p = 1 \text{ or } p = \frac{7}{3}$$

Question 15:

Find the equation of the plane passing through the line of intersection of the planes

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1 \text{ and } \vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$$

and parallel to x-axis.

Answer

The given planes are

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$$

$$\Rightarrow \vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1 = 0$$

$$\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$$

The equation of any plane passing through the line of intersection of these planes is



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$$\begin{aligned} & [\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1] + \lambda [\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4] = 0 \\ & \vec{r} \cdot [(2\lambda + 1)\hat{i} + (3\lambda + 1)\hat{j} + (1 - \lambda)\hat{k}] + (4\lambda + 1) = 0 \quad \dots(1) \end{aligned}$$

Its direction ratios are $(2\lambda + 1)$, $(3\lambda + 1)$, and $(1 - \lambda)$.

The required plane is parallel to x-axis. Therefore, its normal is perpendicular to x-axis.

The direction ratios of x-axis are 1, 0, and 0.

$$\therefore 1 \cdot (2\lambda + 1) + 0(3\lambda + 1) + 0(1 - \lambda) = 0$$

$$\Rightarrow 2\lambda + 1 = 0$$

$$\Rightarrow \lambda = -\frac{1}{2}$$

Substituting $\lambda = -\frac{1}{2}$ in equation (1), we obtain

$$\Rightarrow \vec{r} \cdot \left[-\frac{1}{2}\hat{j} + \frac{3}{2}\hat{k} \right] + (-3) = 0$$

$$\Rightarrow \vec{r} \cdot (\hat{j} - 3\hat{k}) + 6 = 0$$

Therefore, its Cartesian equation is $y - 3z + 6 = 0$

This is the equation of the required plane.

Question 16:

If O be the origin and the coordinates of P be $(1, 2, -3)$, then find the equation of the plane passing through P and perpendicular to OP.

Answer

The coordinates of the points, O and P, are $(0, 0, 0)$ and $(1, 2, -3)$ respectively.

Therefore, the direction ratios of OP are $(1 - 0) = 1$, $(2 - 0) = 2$, and $(-3 - 0) = -3$

It is known that the equation of the plane passing through the point (x_1, y_1, z_1) is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$
 where, a, b, and c are the direction ratios of normal.

Here, the direction ratios of normal are 1, 2, and -3 and the point P is $(1, 2, -3)$. Thus,

the equation of the required plane is



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$$1(x-1)+2(y-2)-3(z+3)=0$$
$$\Rightarrow x+2y-3z-14=0$$

Question 17:

Find the equation of the plane which contains the line of intersection of the planes

$$\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0 \quad \vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$$

$$\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$$

and which is perpendicular to the plane

Answer

The equations of the given planes are

$$\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0 \quad \dots(1)$$

$$\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0 \quad \dots(2)$$

The equation of the plane passing through the line intersection of the plane given in equation (1) and equation (2) is

$$[\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4] + \lambda [\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5] = 0$$

$$\vec{r} \cdot [(2\lambda + 1)\hat{i} + (\lambda + 2)\hat{j} + (3 - \lambda)\hat{k}] + (5\lambda - 4) = 0 \quad \dots(3)$$

The plane in equation (3) is perpendicular to the plane, $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$



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$$\therefore 5(2\lambda + 1) + 3(\lambda + 2) - 6(3 - \lambda) = 0$$

$$\Rightarrow 19\lambda - 7 = 0$$

$$\Rightarrow \lambda = \frac{7}{19}$$

Substituting $\lambda = \frac{7}{19}$ in equation (3), we obtain

$$\Rightarrow \vec{r} \cdot \left[\frac{33}{19}\hat{i} + \frac{45}{19}\hat{j} + \frac{50}{19}\hat{k} \right] - \frac{41}{19} = 0$$

$$\Rightarrow \vec{r} \cdot (33\hat{i} + 45\hat{j} + 50\hat{k}) - 41 = 0 \quad \dots(4)$$

This is the vector equation of the required plane.

The Cartesian equation of this plane can be obtained by substituting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in equation (3).

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (33\hat{i} + 45\hat{j} + 50\hat{k}) - 41 = 0$$

$$\Rightarrow 33x + 45y + 50z - 41 = 0$$

Question 18:

Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line

$$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k}) \quad \text{and the plane} \quad \vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5.$$

Answer

The equation of the given line is

$$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k}) \quad \dots(1)$$

The equation of the given plane is

$$\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5 \quad \dots(2)$$

Substituting the value of \vec{r} from equation (1) in equation (2), we obtain



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$$\begin{aligned} & [2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})] \cdot (\hat{i} - \hat{j} + \hat{k}) = 5 \\ \Rightarrow & [(3\lambda + 2)\hat{i} + (4\lambda - 1)\hat{j} + (2\lambda + 2)\hat{k}] \cdot (\hat{i} - \hat{j} + \hat{k}) = 5 \\ \Rightarrow & (3\lambda + 2) - (4\lambda - 1) + (2\lambda + 2) = 5 \\ \Rightarrow & \lambda = 0 \end{aligned}$$

Substituting this value in equation (1), we obtain the equation of the line as

$$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}$$

This means that the position vector of the point of intersection of the line and the plane

$$\text{is } \vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}$$

This shows that the point of intersection of the given line and plane is given by the coordinates, (2, -1, 2). The point is (-1, -5, -10).

The distance d between the points, (2, -1, 2) and (-1, -5, -10), is

$$d = \sqrt{(-1-2)^2 + (-5+1)^2 + (-10-2)^2} = \sqrt{9+16+144} = \sqrt{169} = 13$$

Question 19:

Find the vector equation of the line passing through (1, 2, 3) and parallel to the planes

$$\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5 \text{ and } \vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$$

Answer

Let the required line be parallel to vector \vec{b} given by,

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

The position vector of the point (1, 2, 3) is $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

The equation of line passing through (1, 2, 3) and parallel to \vec{b} is given by,

$$\begin{aligned} \vec{r} &= \vec{a} + \lambda\vec{b} \\ \Rightarrow \vec{r} &= (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) \quad \dots(1) \end{aligned}$$

The equations of the given planes are



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$$\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5 \quad \dots(2)$$

$$\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6 \quad \dots(3)$$

The line in equation (1) and plane in equation (2) are parallel. Therefore, the normal to the plane of equation (2) and the given line are perpendicular.

$$\Rightarrow (\hat{i} - \hat{j} + 2\hat{k}) \cdot \lambda (b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) = 0$$

$$\Rightarrow \lambda (b_1 - b_2 + 2b_3) = 0$$

$$\Rightarrow b_1 - b_2 + 2b_3 = 0 \quad \dots(4)$$

$$\text{Similarly, } (3\hat{i} + \hat{j} + \hat{k}) \cdot \lambda (b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) = 0$$

$$\Rightarrow \lambda (3b_1 + b_2 + b_3) = 0$$

$$\Rightarrow 3b_1 + b_2 + b_3 = 0 \quad \dots(5)$$

From equations (4) and (5), we obtain

$$\frac{b_1}{(-1) \times 1 - 1 \times 2} = \frac{b_2}{2 \times 3 - 1 \times 1} = \frac{b_3}{1 \times 1 - 3(-1)}$$

$$\Rightarrow \frac{b_1}{-3} = \frac{b_2}{5} = \frac{b_3}{4}$$

Therefore, the direction ratios of \vec{b} are $-3, 5,$ and $4.$

$$\therefore \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = -3\hat{i} + 5\hat{j} + 4\hat{k}$$

Substituting the value of \vec{b} in equation (1), we obtain

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda (-3\hat{i} + 5\hat{j} + 4\hat{k})$$

This is the equation of the required line.

Question 20:

Find the vector equation of the line passing through the point $(1, 2, -4)$ and

perpendicular to the two lines: $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$



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Answer

Let the required line be parallel to the vector \vec{b} given by, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

The position vector of the point $(1, 2, -4)$ is $\vec{a} = \hat{i} + 2\hat{j} - 4\hat{k}$

The equation of the line passing through $(1, 2, -4)$ and parallel to vector \vec{b} is

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

$$\Rightarrow \vec{r}(\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) \quad \dots(1)$$

The equations of the lines are

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \quad \dots(2)$$

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5} \quad \dots(3)$$

Line (1) and line (2) are perpendicular to each other.

$$\therefore 3b_1 - 16b_2 + 7b_3 = 0 \quad \dots(4)$$

Also, line (1) and line (3) are perpendicular to each other.

$$\therefore 3b_1 + 8b_2 - 5b_3 = 0 \quad \dots(5)$$

From equations (4) and (5), we obtain

$$\frac{b_1}{(-16)(-5) - 8 \times 7} = \frac{b_2}{7 \times 3 - 3(-5)} = \frac{b_3}{3 \times 8 - 3(-16)}$$

$$\Rightarrow \frac{b_1}{24} = \frac{b_2}{36} = \frac{b_3}{72}$$

$$\Rightarrow \frac{b_1}{2} = \frac{b_2}{3} = \frac{b_3}{6}$$

∴ Direction ratios of \vec{b} are 2, 3, and 6.

$$\therefore \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

Substituting $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ in equation (1), we obtain

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

This is the equation of the required line.



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Question 21:

Prove that if a plane has the intercepts a , b , c and is at a distance of P units from the

origin, then $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$

Answer

The equation of a plane having intercepts a , b , c with x , y , and z axes respectively is given by,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \quad \dots(1)$$

The distance (p) of the plane from the origin is given by,

$$p = \frac{\left| \frac{0}{a} + \frac{0}{b} + \frac{0}{c} - 1 \right|}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}}$$

$$\Rightarrow p = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$$

$$\Rightarrow p^2 = \frac{1}{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}$$

$$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$$

Question 22:

Distance between the two planes: $2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$ is

- (A) 2 units (B) 4 units (C) 8 units (D) $\frac{2}{\sqrt{29}}$ units

Answer

The equations of the planes are



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$$2x + 3y + 4z = 4 \quad \dots(1)$$

$$4x + 6y + 8z = 12$$

$$\Rightarrow 2x + 3y + 4z = 6 \quad \dots(2)$$

It can be seen that the given planes are parallel.

It is known that the distance between two parallel planes, $ax + by + cz = d_1$ and $ax + by + cz = d_2$, is given by,

$$D = \left| \frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}} \right|$$

$$\Rightarrow D = \left| \frac{6 - 4}{\sqrt{(2)^2 + (3)^2 + (4)^2}} \right|$$

$$D = \frac{2}{\sqrt{29}}$$

Thus, the distance between the lines is $\frac{2}{\sqrt{29}}$ units.

Hence, the correct answer is D.

Question 23:

The planes: $2x - y + 4z = 5$ and $5x - 2.5y + 10z = 6$ are

(A) Perpendicular (B) Parallel (C) intersect y-axis (D) passes through $\left(0, 0, \frac{5}{4}\right)$

Answer

The equations of the planes are

$$2x - y + 4z = 5 \quad \dots (1)$$

$$5x - 2.5y + 10z = 6 \quad \dots (2)$$

It can be seen that,



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$$\frac{a_1}{a_2} = \frac{2}{5}$$

$$\frac{b_1}{b_2} = \frac{-1}{-2.5} = \frac{2}{5}$$

$$\frac{c_1}{c_2} = \frac{4}{10} = \frac{2}{5}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, the given planes are parallel.

Hence, the correct answer is B.

