

Matrices

Mathematics XIII



Focus On Free Education

Chapter 3

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(Chapter 3)(Matrices)

(Class – XII)

Exercise 3.1

Question 1:

In the matrix $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & 5/2 & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$, write:

- (i) The order of the matrix
- (ii) The number of elements,
- (iii) Write the elements a_{13} , a_{21} , a_{33} , a_{24} , a_{23}

Answer 1:

- (i) In the given matrix, the number of rows is 3 and the number of columns is 4. Therefore, the order of the matrix is 3×4 .
- (ii) Since the order of the matrix is 3×4 , there are $3 \times 4 = 12$ elements in it.
- (iii) $a_{13} = 19$, $a_{21} = 35$, $a_{33} = -5$, $a_{24} = 12$, $a_{23} = \frac{5}{2}$

Question 2:

If a matrix has 24 elements, what are the possible order it can have? What, if it has 13 elements?

Answer 2:

We know that if a matrix is of the order $m \times n$, it has mn elements. Thus, to find all the possible orders of a matrix having 24 elements, we have to find all the ordered pairs of natural numbers whose product is 24.

The ordered pairs are: (1, 24), (24, 1), (2, 12), (12, 2), (3, 8), (8, 3), (4, 6), and (6, 4)

Hence, the possible orders of a matrix having 24 elements are:

1×24 , 24×1 , 2×12 , 12×2 , 3×8 , 8×3 , 4×6 , and 6×4

(1, 13) and (13, 1) are the ordered pairs of natural numbers whose product is 13.

Hence, the possible orders of a matrix having 13 elements are 1×13 and 13×1 .

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Question 3:

If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

Answer 3:

We know that if a matrix is of the order $m \times n$, it has mn elements. Thus, to find all the possible orders of a matrix having 18 elements, we have to find all the ordered pairs of natural numbers whose product is 18.

The ordered pairs are: (1, 18), (18, 1), (2, 9), (9, 2), (3, 6), and (6, 3)

Hence, the possible orders of a matrix having 18 elements are:

1×18 , 18×1 , 2×9 , 9×2 , 3×6 , and 6×3

(1, 5) and (5, 1) are the ordered pairs of natural numbers whose product is 5.

Hence, the possible orders of a matrix having 5 elements are 1×5 and 5×1 .

Question 4:

Construct a 2×2 matrix, $A = [a_{ij}]$, whose elements are given by:

(i) $a_{ij} = \frac{(i+j)^2}{2}$

(ii) $a_{ij} = \frac{i}{j}$

(iii) $a_{ij} = \frac{(i+2j)^2}{2}$

Answer 4:

(i) Given that $a_{ij} = \frac{(i+j)^2}{2}$

In general a 2×2 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

So,

$$a_{11} = \frac{(1+1)^2}{2} = \frac{4}{2} = 2$$

$$a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2}$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}$$

$$a_{22} = \frac{(2+2)^2}{2} = \frac{16}{2} = 8$$

Therefore, the required matrix is $A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$

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(ii) $a_{ij} = \frac{i}{j}$

In general a 2×2 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

So,

$$a_{11} = \frac{1}{1} = 1$$

$$a_{12} = \frac{1}{2}$$

$$a_{21} = \frac{2}{1} = 2$$

$$a_{22} = \frac{2}{2} = 1$$


Therefore, the required matrix is $A = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$

(iii) Given that $a_{ij} = \frac{(i+2j)^2}{2}$

In general a 2×2 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

So,

$$a_{11} = \frac{(1 + 2 \times 1)^2}{2} = \frac{9}{2}$$


$$a_{12} = \frac{(1 + 2 \times 2)^2}{2} = \frac{25}{2}$$

$$a_{21} = \frac{(2 + 2 \times 1)^2}{2} = \frac{16}{2} = 8$$

$$a_{22} = \frac{(2 + 2 \times 2)^2}{2} = \frac{36}{2} = 18$$

Therefore, the required matrix is $A = \begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$

Question 5:

Construct a 3×4 matrix, whose elements are given by

(i) $a_{ij} = \frac{1}{2}|-3i + j|$

(ii) $a_{ij} = 2i - j$

Answer 5:

(i) $a_{ij} = \frac{1}{2}|-3i + j|$

In general a 3×4 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$

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$$a_{11} = \frac{1}{2} |-3 \times 1 + 1| = 1$$

$$a_{12} = \frac{1}{2} |-3 \times 1 + 2| = \frac{1}{2}$$

$$a_{13} = \frac{1}{2} |-3 \times 1 + 3| = 0$$

$$a_{14} = \frac{1}{2} |-3 \times 1 + 4| = \frac{1}{2}$$

$$a_{21} = \frac{1}{2} |-3 \times 2 + 1| = \frac{5}{2}$$

$$a_{22} = \frac{1}{2} |-3 \times 2 + 2| = 2$$

$$a_{23} = \frac{1}{2} |-3 \times 2 + 3| = \frac{3}{2}$$

$$a_{24} = \frac{1}{2} |-3 \times 2 + 4| = 1$$



$$a_{31} = \frac{1}{2} |-3 \times 3 + 1| = 4$$

$$a_{32} = \frac{1}{2} |-3 \times 3 + 2| = \frac{7}{2}$$

$$a_{33} = \frac{1}{2} |-3 \times 3 + 3| = 3$$

$$a_{34} = \frac{1}{2} |-3 \times 3 + 4| = \frac{5}{2}$$

Therefore, the required matrix is $A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

(ii) $a_{ij} = 2i - j$

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In general a 3×4 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$

$$a_{11} = 2 \times 1 - 1 = 1$$

$$a_{12} = 2 \times 1 - 2 = 0$$

$$a_{13} = 2 \times 1 - 3 = -1$$

$$a_{14} = 2 \times 1 - 4 = -2$$

$$a_{21} = 2 \times 2 - 1 = 3$$

$$a_{22} = 2 \times 2 - 2 = 2$$

$$a_{23} = 2 \times 2 - 3 = 1$$

$$a_{24} = 2 \times 2 - 4 = 0$$

$$a_{31} = 2 \times 3 - 1 = 5$$

$$a_{32} = 2 \times 3 - 2 = 4$$

$$a_{33} = 2 \times 3 - 3 = 3$$

$$a_{34} = 2 \times 3 - 4 = 2$$



Therefore, the required matrix is $A = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$

Question 6:

Find the value of x , y , and z from the following equation:

$$(i) \begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix} \quad (ii) \begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix} \quad (iii) \begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

Answer 6:

$$(i) \begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$$

As the given matrices are equal, their corresponding elements are also equal.

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Comparing the corresponding elements, we get:

$$x = 1, y = 4, \text{ and } z = 3$$

$$(ii) \begin{bmatrix} x + y & 2 \\ 5 + z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

As the given matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we

$$\text{get: } x + y = 6, xy = 8, 5 + z = 5$$

$$\text{Now, } 5 + z = 5 \Rightarrow z = 0$$

we know that:

$$(x - y)^2 = (x + y)^2 - 4xy$$

$$\Rightarrow (x - y)^2 = 36 - 32 = 4$$

$$\Rightarrow x - y = \pm 2$$

Now, when $x - y = 2$ and $x + y = 6$, we get $x = 4$ and $y = 2$

When $x - y = -2$ and $x + y = 6$, we get $x = 2$ and $y = 4$

$\therefore x = 4, y = 2, \text{ and } z = 0$ or $x = 2, y = 4, \text{ and } z = 0$

$$(iii) \begin{bmatrix} x + y + z \\ x + z \\ y + z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

As the two matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

$$x + y + z = 9 \dots (1)$$

$$x + z = 5 \dots\dots\dots (2)$$

$$y + z = 7 \dots\dots\dots (3)$$

From (1) and (2), we have:

$$y + 5 = 9$$

$$\Rightarrow y = 4$$

Then, from (3), we have:

$$4 + z = 7$$

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$$\Rightarrow z = 3$$

$$\therefore x + z = 5$$

$$\Rightarrow x = 2$$

$$\therefore x = 2, y = 4 \text{ and } z = 3.$$

Question 7:

Find the value of a , b , c , and d from the equation:

$$\begin{bmatrix} a - b & 2a + c \\ 2a - b & 3c + d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

Answer 7:

$$\begin{bmatrix} a - b & 2a + c \\ 2a - b & 3c + d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

As the two matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

$$a - b = -1 \dots\dots (1)$$

$$2a - b = 0 \dots\dots (2)$$

$$2a + c = 5 \dots\dots (3)$$

$$3c + d = 13 \dots\dots (4)$$



From (2), we have:

$$b = 2a$$

Then, from (1), we have:

$$a - 2a = -1 \quad \Rightarrow a = 1 \quad \Rightarrow b = 2$$

Now, from (3), we have:

$$2 \times 1 + c = 5 \quad \Rightarrow c = 3$$

From (4) we have:

$$3 \times 3 + d = 13$$

$$\Rightarrow 9 + d = 13 \quad \Rightarrow d = 4$$

$$\therefore a = 1, b = 2, c = 3 \text{ and } d = 4.$$

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Question 8:

$A = [a_{ij}]_{m \times n}$ is a square matrix, if

- (A) $m < n$ (B) $m > n$ (C) $m = n$ (D) None of these

Answer 8:

The correct answer is C.

It is known that a given matrix is said to be a square matrix if the number of rows is equal to the number of columns.

Therefore, $A = [a_{ij}]_{m \times n}$ is a square matrix, if $m = n$.

Question 9:

Which of the given values of x and y make the following pair of matrices equal

$$\begin{bmatrix} 3x + 7 & 5 \\ y + 1 & 2 - 3x \end{bmatrix} = \begin{bmatrix} 0 & y - 2 \\ 8 & 4 \end{bmatrix}$$

(A) $x = \frac{-1}{3}, y = 7$

(C) $y = 7, x = \frac{-2}{3}$



(B) Not possible to find

(D) $x = \frac{-1}{3}, y = \frac{-2}{3}$

Answer 9:

The correct answer is B.

It is given that $\begin{bmatrix} 3x + 7 & 5 \\ y + 1 & 2 - 3x \end{bmatrix} = \begin{bmatrix} 0 & y - 2 \\ 8 & 4 \end{bmatrix}$

Equating the corresponding elements, we get:

$$3x + 7 = 0 \Rightarrow x = \frac{-7}{3} \quad \text{and} \quad 5 = y - 2 \Rightarrow y = 7$$

$$y + 1 = 8 \Rightarrow y = 7 \quad \text{and} \quad 2 - 3x = 4 \Rightarrow x = \frac{-2}{3}$$

We find that on comparing the corresponding elements of the two matrices, we get two different values of x , which is not possible.

Hence, it is not possible to find the values of x and y for which the given matrices are equal.

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Question 10:

The number of all possible matrices of order 3×3 with each entry 0 or 1 is:

- (A) 27 (B) 18 (C) 81 (D) 512

Answer 10:

The correct answer is D.

The given matrix of the order 3×3 has 9 elements and each of these elements can be either 0 or 1.

Now, each of the 9 elements can be filled in two possible ways.

Therefore, by the multiplication principle, the required number of possible matrices is $2^9 = 512$

